Temperature induced phase averaging in one-dimensional mesoscopic systems

Severin G. Jakobs,[1] Volker Meden,[2] Herbert Schoeller,[1] and Tilman Enss[3]

[1]*Institut für Theoretische Physik A, RWTH Aachen, D-52056 Aachen, Germany*
[2]*Institut für Theoretische Physik, Universität Göttingen, D-37077 Göttingen, Germany*
[3]*Dipartimento di Fisica, Università di Roma "La Sapienza", Piazzale Aldo Moro 2, I-00185 Roma, Italy*
(Dated: April 22, 2018)

We analyse phase averaging in one-dimensional interacting mesoscopic systems with several barriers and show that for incommensurate positions an independent average over several phases can be induced by finite temperature. For three strong barriers with conductances G_i and mutual distances larger than the thermal length, we obtain $G \sim \sqrt{G_1 G_2 G_3}$ for the total conductance G. For an interacting wire, this implies power laws in $G(T)$ with novel exponents, which we propose as an experimental fingerprint to distinguish temperature induced phase averaging from dephasing.

PACS numbers: 71.10.Pm, 72.10.Fk, 73.63.Nm

I. INTRODUCTION

Mesoscopic systems are characterized by spatial dimensions smaller than the phase breaking length L_φ so that the phase of an electron is not destroyed by inelastic processes. Along their optical paths, electrons pick up phases from propagation and scattering. Even for negligible dephasing, certain circumstances can lead to an *averaging* of these phases, a phenomenon which has been analyzed in connection with localization, see e.g. Refs. 1,2. It remains however a fundamental task to compare in detail the effects of phase averaging and dephasing. For a noninteracting model system it was recently shown that a single one-channel dephasing probe gives the same full counting statistics as a single phase-averaging probe.[3] However, it was noted that this result does no longer hold for several probes, i.e. phase averaging over many independent phases seems to be fundamentally different from dephasing. In the present paper we address this issue in detail. We discuss a generic one-dimensional interacting system coupled to two reservoirs with few barriers at arbitrary but fixed positions. We show that for incommensurate barrier positions an independent average over several phases can be induced by finite temperature. For more than two barriers we find that phase averaging drastically differs from dephasing and show that the commonly made assumption that barriers with mutual distances larger than the thermal length $L_T = v_F/T$ can be treated independently[4,5,6] (note that in experiment contacts to leads constitute additional barriers) does not hold in the absence of dephasing. This means that even in the high-temperature regime interference effects are still important, as also shown recently for quantum dots in the sequential tunneling regime.[7,8] For three strong barriers with individual conductances G_i, we obtain $G \sim \sqrt{G_1 G_2 G_3}$ for the total conductance, in drastic contrast to the addition of resistances which follows from dephasing. For an interacting quantum wire (QW) this implies power-law scaling of $G(T)$ with novel exponents which can be used as an experimental fingerprint for phase averaging.

In section II we introduce a noninteracting model for a quantum wire with barriers which allows a simple discussion of temperature induced phase averaging. In section III we study the influence of temperature induced phase averaging on the scaling beghaviour of interacting quantum wires and propose an experimental setup to distinguish phase averaging from dephasing. While sections II and III focus primarily on a wire with three barriers, section IV treats phase averaging for a wire with four barriers.

II. NONINTERACTING WIRE

For a discussion of the basic physical idea consider first the noninteracting case. We model the QW coupled to leads by an infinite tight-binding chain for spinless electrons at half-filling ($\mu = 0$)

$$H = - \sum_{n \in \mathbb{Z}, n \neq n_i} c_{n+1}^\dagger c_n - \sum_{i=1}^{p} \tau_i \, c_{n_i+1}^\dagger c_{n_i} + \text{h.c.} \quad (1)$$

with hopping matrix elements equal to 1 (defining the energy unit) except for $n = n_i$, $i = 1, \ldots, p$, where the barriers are situated. Without barriers the dispersion relation is $\epsilon = -2 \cos{(ka)}$ with band width $D = 4$, where k is the momentum and a the lattice spacing. The distance between subsequent barriers is $L_i = a N_i$ with $N_i = n_{i+1} - n_i$. We characterize the barriers by their transmission $t_i(\epsilon)$ and reflection $r_i^\pm(\epsilon) = |r_i(\epsilon)| e^{i \delta_i^\pm(\epsilon)}$ amplitudes for right- and left-running scattering waves. The linear conductance in units of e^2/h follows from[9]

$$G(T) = \int d\epsilon \left(-\frac{\partial f}{\partial \epsilon} \right) |t(\epsilon)|^2 \ . \quad (2)$$

where $f = 1/(e^{\epsilon/T} + 1)$, and $t(\epsilon)$ is the transmission amplitude. Thus, the effect of temperature is an average of the transmission probability $|t|^2$ over an energy range $\Delta \epsilon \sim T$. This is fundamentally different from dephasing which destroys the phase information for each

individual path of the electron. If one takes account of dephasing by summing up the classical probabilities of all paths, one obtains $|r|^2 = \prod_i |r_i|^2 + \sum_i |t_i|^2$, with $|r|^2 = 1 - |t|^2$.[10] Since $|r|^2/|t|^2$ is proportional to the resistance of a barrier, this gives the classical law of adding up resistances in series. In contrast, we now show that temperature induced energy averaging leads to completely different results. Since $|t_i(\epsilon)|$ and $|r_i(\epsilon)|$ vary slowly with energy on the scale $T \ll D$, temperature averages only over the rapidly varying phases an electron acquires by bouncing back and forth between two subsequent barriers $\varphi_i(\epsilon) = 2k(\epsilon)L_i + \delta_i^-(\epsilon) + \delta_{i+1}^+(\epsilon)$. As $\delta_i^{\pm}$ depends weakly on energy, the change of φ_i over the energy range $\Delta\epsilon \sim T$ near the Fermi energy is given by $\Delta\varphi_i \sim \Delta k L_i \sim \frac{\Delta\epsilon}{\partial\epsilon/\partial k} L_i \sim L_i/L_T$. Thus, for $L_T \sim L_i$, $\Delta\varphi_i$ is roughly given by 2π. For all $L_T \ll L_i$ and incommensurate length L_i (i.e., $m_iL_j = m_jL_i$ is only valid for large coprime integers $m_{i,j}$), this implies that the average over energy will cover a representative part of the multidimensional phase space defined by $(\varphi_1, \ldots, \varphi_{p-1})$ (see Fig. 1). In this case, temperature induced energy averaging is equivalent to an *independent* average over all phases φ_i, and we can replace $|t|^2$ in Eq. (2) by the phase-averaged quantity

$$\langle |t|^2 \rangle = \prod_{i=1}^{p-1} \frac{1}{2\pi} \int_0^{2\pi} d\varphi_i \, |t|^2 \qquad (3)$$

provided that $v_F/L_i \ll T \ll D$. Note that this result does not hold for commensurate lengths L_i (where the path in phase space closes very quickly; see below) and becomes less relevant for a large number p of barriers (since L_i/L_T has to be chosen too large in order to cover a representative part of the high dimensional phase space).

The phase average can be calculated easily for a small number of barriers. For two barriers we use $t = t_1t_2/(1 - e^{i\varphi_1}|r_1r_2|)$ and for three barriers (forming two dots which we refer to as left and right)

$$t = t_1t_2t_3 \big[\big(1 - e^{i\varphi_L}|r_1r_2| \big) \big(1 - e^{i\varphi_R}|r_2r_3| \big) + e^{i(\varphi_L + \varphi_R)}|r_1r_3||t_2|^2 \big]^{-1}, \quad (4)$$

with $\varphi_{L,R} \equiv \varphi_{1,2}$. By accident, phase averaging yields the addition of resistances for two barriers. In contrast, for three barriers we obtain

$$\langle |t|^2 \rangle = \frac{T_1T_2T_3}{\sqrt{(\sum_{i<j} T_iT_j)^2 + 4T_1T_2T_3(1 - \sum_i T_i)}}, \quad (5)$$

with $T_i = |t_i|^2$. For weak barriers, $T_i \approx 1$, this gives $\langle |t|^2 \rangle \approx T_1T_2T_3$ leading to exponentially small transmission when scaling it up to many barriers, in analogy to Ref. 1. For strong barriers, $T_i \ll 1$, we obtain the surprising result

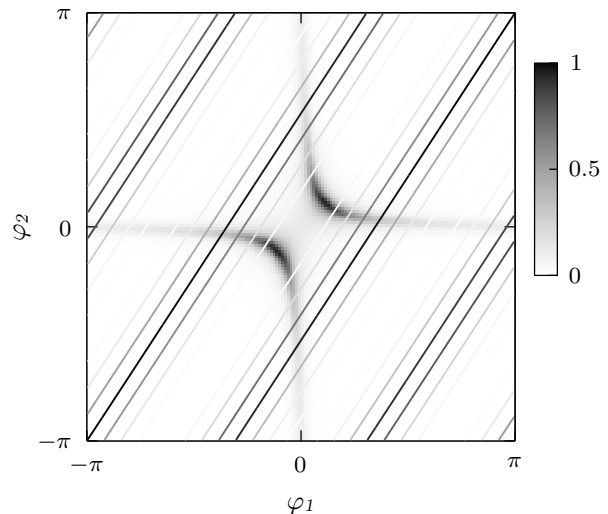$$\langle |t|^2 \rangle \approx \frac{1}{2}\sqrt{T_1T_2T_3}, \qquad (6)$$

FIG. 1: Density plot of the transmission probability $|t(\varphi_1, \varphi_2)|^2$ as function of the phases $\varphi_{1,2}$ for a noninteracting QW with three barriers $\tau_{1,2,3} = 0.35$. Lines: Path which the integral in Eq. (2) traces in the (φ_1, φ_2)-plane for $N_1 = 4940$, $N_2 = 7560$. The grayscale of the line indicates the relative amplitude of $-\partial f/\partial\epsilon$ for $T = 1.5 \cdot 10^{-3}$, the same grayscale applies as for the density plot. For sufficiently high temperature the line covers a representative part of the plane.

leading to $G \sim \sqrt{G_1G_2G_3}$ when inserted in Eq. (2). Fig. 2 shows that the phase-averaged G through three barriers at incommensurate positions agrees precisely with the exact one for sufficiently large T, whereas summing up the individual resistances is incorrect for all T. In the inset of Fig. 2 we present the energy dependence of the total transmission probability. It forms energetically large superstructures, as the height and area of the peaks are very sensitive to the mutual distance of the resonance positions of the left and right dot. Those are roughly given by even (odd) multiples of $\pi/(N_{L,R} + 1)$ for odd (even) $N_{L,R}$. Thus, if $m_R(N_L + 1) = m_L(N_R + 1)$ is valid only for large coprime integers $m_{L,R}$ (incommensurate case[11]), the mutual energetic distance of left- and right-dot resonance peaks of $|t|^2$ shifts slowly along the energy axis, leading to a modulation of the peak height and area. This provides another picture for temperature induced energy averaging: If T is large enough to average over a sufficient part of a superstructure, it averages over many mutual distances between left- and right-dot resonances, which is equivalent to phase averaging.

In contrast, for dots of commensurate length, i.e., when $m_R(N_L + 1) = m_L(N_R + 1)$ is fulfilled for small coprime integers $m_{L,R}$, every m_Lth left-dot and m_Rth right-dot resonance coincide, whereas the other resonances are well separated in energy (see inset in Fig. 3). It can be shown that coinciding resonances (avoided crossings) give rise to two peaks of the total transmission probability, each one with area $A = 4\pi\big[(N_L+1)/\tau_1^2 + (N_R+1)/\tau_3^2\big]^{-1} = \mathcal{O}(\tau^2)$. Note that while τ_2 is proportional to the distance of the two peaks, it does not influence their area. The well-
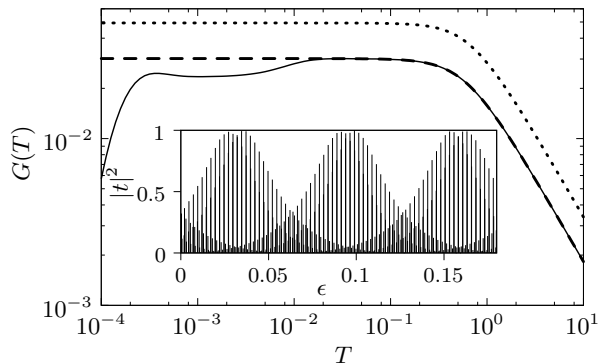
FIG. 2: Linear conductance as function of temperature for a noninteracting QW with three barriers $\tau_{1,2,3} = 0.2$ and incommensurate dot length, $N_L = 4980$, $N_R = 7520$. The solid line corresponds to Eq. (2). For higher T, the dashed line which results from using the phase averaged transmission given by Eq. (5) in Eq. (2) matches perfectly. Summing up the individual resistances (dotted line) yields an incorrect result. *Inset:* $|t|^2$ as a function of energy. The changing mutual energetic distance of resonances of the left and right dot leads to energetically large superstructures of the total transmission probability.
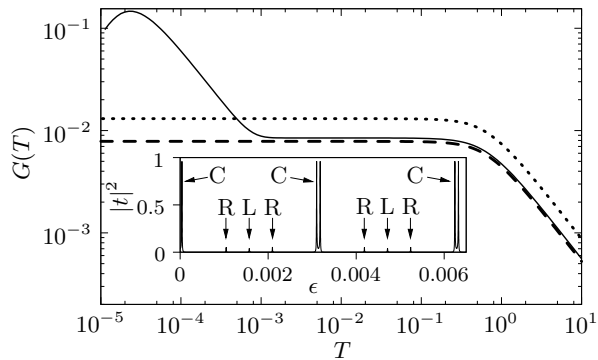

FIG. 3: Linear conductance as function of temperature for a noninteracting QW with three barriers $\tau_{1,2,3} = 0.1$ and commensurate dot length, $N_L + 1 = 4000$, $N_R + 1 = 6000$, hence $m_L = 2, m_R = 3$. The solid line results from Eq. (2). For temperatures greater than the distance of coinciding peaks (compare inset), the dashed line corresponding to Eq. (7) is a valid approximation. Summing up the individual resistances (dotted line) yields an incorrect result. *Inset:* $|t|^2$ as function of energy. Labels indicate well-separated left-dot (L) and right-dot (R) resonances as well as coinciding (C) ones. The distance of the coinciding resonances is $\Delta\epsilon_C \approx \frac{2\pi m_L}{N_L+1} = \frac{2\pi m_R}{N_R+1} \approx \frac{\pi}{1000}$.

separated peaks can be neglected when calculating the total conductance through the QW because they have a much smaller area of $\mathcal{O}\left(\tau^4\right)$ than the coinciding peaks and they are not in great majority (as $m_{L,R}$ are small). For $T \gtrsim 2\pi \frac{m_L}{N_L+1} = 2\pi \frac{m_R}{N_R+1}$, the energy-integral in Eq. (2) averages over the large coinciding peaks. For

the total conductance it follows (see Fig. 3)

$$\frac{1}{G} = \frac{m_L}{G_1} + \frac{m_R}{G_3} \tag{7}$$

with $G_i = 4\tau_i^2$. Since the smaller peaks have been neglected, this formula slightly underestimates G, but becomes more and more accurate for stronger barriers. Eq. (7) in obviously inconsistent with summing up the single resistances, and the total conductance is independent of the strength of the barrier in the middle.

III. INTERACTING WIRE

We now turn to the more realistic case of an interacting QW (Luttinger liquid, LL) on sites $n = 1, \ldots, N$ by adding a short ranged interaction $H_{int} = \sum_{n=1}^{N-1} U_n \rho_n \rho_{n+1}$ with $\rho_n = c_n^\dagger c_n - \frac{1}{2}$. Abrupt contacts to the leads follow if the interaction is chosen to be the same on all sites, $U_n \equiv U$. For smooth contacts U_n rises slowly from zero at the leads to its full value within the wire over roughly 100 sites. The interaction is treated using the truncated functional renormalization group.[12,13] This approach leads to a real and frequency independent self-energy, which then serves as an effective single-particle potential so that Eq. (2) is still applicable. Inelastic processes mediated by the interaction are neglected as they would just contribute subleading corrections.

A single strong impurity in a LL generates a slowly decaying oscillatory effective potential of range L_T which makes the local density of states in the vicinity of a large impurity scale as $\rho(\epsilon = 0) \sim T^{\alpha_B}$.[14,15] The boundary exponent α_B can be computed from the LL parameter K which in turn is known from Bethe ansatz[16], $\alpha_B = \frac{1}{K} - 1 = \frac{2}{\pi} \arcsin(\frac{U}{2})$ at half-filling. For $U = 1$, as taken in the following, our approach is known to produce the exponent $\alpha_B' \approx 0.35$ in good agreement with $\alpha_B = 1/3$.[12]

If a strong barrier is placed within the interacting part of the QW, the density of states scales with T on both sides of the barrier yielding a power-law $G \sim T^{2\alpha_B}$. If a strong barrier separates an interacting part of the wire from a noninteracting part as do tunneling barriers to the leads, $G \sim T^{\alpha_B}$. We are interested in possible power laws if three barriers of mixed types are present.

Given three barriers in the wire, we find that the effective potential formed at one barrier is not affected by the presence of the other barriers, as long as $L_i \gg L_T$. Consequently, the scaling law of the transmission probabilities of the individual barriers $|t_i|^2 \sim \max\{T, |\epsilon|\}^{\alpha_i}$ (with $\alpha_i = \alpha_B$ or $2\alpha_B$ depending on the type of the barrier) is not affected by the presence of the other barriers and thus $G_i(T) \sim T^{\alpha_i}$. Interpreting a barrier and its surrounding oscillations as an effective barrier, we can use Eq. (4) and the picture of two dots in series developed for the noninteracting case. Thus, for incommensurate L_i and sufficiently large T, phase averaging as given by Eq. (3) can as well be applied to an interacting wire (see Fig. 4). This implies power-law scaling of $G \sim \sqrt{G_1 G_2 G_3}$
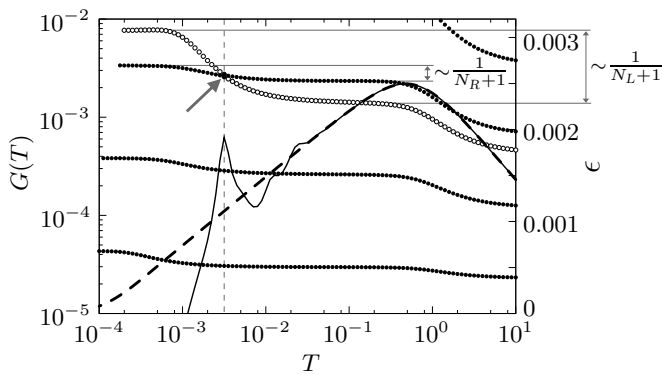
FIG. 4: $G(T)$ (lines) and position of resonance energies of $|t(\epsilon)|^2$ (circles) for $U = 1$ and barriers at the two contacts and a barrier within the wire, $\tau_{1,2,3} = 0.1$, $N_L = 1800$, $N_R = 8200$. The solid line corresponds to Eq. (2). For higher temperatures, the dashed line which results from using the phase averaged transmission given by Eq. (5) in Eq. (2) matches perfectly. As left-dot resonances (open circles) are shifted a different amount in energy with changing temperature than right-dot resonances (filled circles), an avoided crossing can occur (arrow) leading to a drastically enhanced conductance if temperature is low.
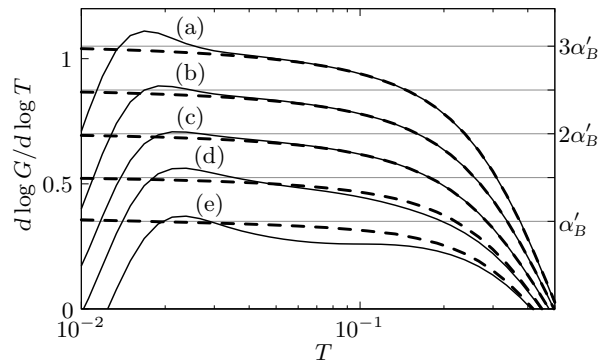

FIG. 5: Local exponent of $G(T)$ for $U = 1$ and three barriers, $\tau_{1,2,3} = 0.1$, $N_L = 4930$, $N_R = 5070$. (a) All three barriers within the wire. (b) One barrier at end-contact to lead, two barriers within the wire. (c) Two barriers at end-contacts, one barrier within the wire. (d) One barrier at bulk-contact, one barrier at end-contact, one barrier within the wire. (e) Two barriers at bulk-contacts, one barrier within the wire. The solid lines correspond to Eq. (2), the dashed lines result from using the phase averaged transmission given by Eq. (5) in Eq. (2). The different levels of the exponents (multiples of $\alpha'_B/2$) are clearly visible. In the bulk-contacted cases (d) and (e), phase averaging is not as good an approximation as in the other cases.

where the exponent depends on *all* exponents of the individual barriers

$$G(T) \sim T^{\frac{1}{2}(\alpha_1 + \alpha_2 + \alpha_3)}. \tag{8}$$

Therefore, combinations of different barriers will produce exponents equal to different integer multiples of $\alpha_B/2$. For three barriers within the interacting part of the wire ($\alpha_{1,2,3} = 2\alpha_B$), Eq. (8) yields $G \sim T^{3\alpha_B}$. If one of the three barriers is a contact to a noninteracting lead, it follows $G \sim T^{5\alpha_B/2}$. Two contacts and one barrier within the interacting part imply $G \sim T^{2\alpha_B}$ (see Fig. 5).

In addition, in Fig. 5 we show the effect of bulk contacts (in contrast to end contacts discussed so far). Also for this setup $G(T)$ at sufficiently large T is given approximately by $G \sim \sqrt{G_1 G_2 G_3}$. Since bulk contacts do not change significantly the density of states in the wire[17], their contribution to the exponent in Eq. (8) is $\alpha_i = 0$. This provides a concrete experimental setup to measure fingerprints of phase averaging by considering a quantum wire with a single impurity contacted by two end contacts and, in situ, also by two scanning tunneling microscopy contacts situated left and right to the impurity but far away from the end contacts compared to L_T. By either measuring transport through the two end contacts, or one end and one bulk contact, or two bulk contact, the situation (c), (d), and (e) of Fig. 5 are realized, respectively. If phase averaging dominates different exponents will be observed, whereas in the case of dominant dephasing the exponent will always be given by the impurity exponent $2\alpha_B$.[18]

An additional feature in Fig. 4 is the occurrence of peaks in $G(T)$ at certain moderate temperatures. They arise when left- and right-dot resonances coincide lead-

ing to an avoided crossing with a strongly enhanced total transmission. As the transmission and reflection amplitudes of the single effective barriers depend on T via the range L_T of the oscillations of the effective potential, the dot resonances shift energetically with T. The peak shifts occur in two steps, when temperature is lowered from far larger than the bandwidth to zero: The first step occurs when T sweeps over the largest part of the band and is due to a renormalization of the hopping in the chain. The second step occurs when T sweeps over the very peak under consideration. The size in energy of this second step is proportional to the level spacing of the corresponding dot and therefore in general different for left- and right-dot resonances making possible an avoided crossing (see Fig. 4). At higher temperatures, there are so many transmission resonances contributing to the conductance that the enlargement of single peaks due to their avoided crossing is not visible in the conductance.

The temperature dependence of the dot resonances has also the consequence that only two dots of exactly the same length represent a truly commensurate configuration because only for $L_L = L_R$ regularly coinciding left- and right-dot resonances occur at every temperature. In this special case our data confirms that Eq. (7), which now reads $G^{-1} = G_1^{-1} + G_3^{-1}$, is generalizable to interacting QWs. Depending on whether the single barriers are within the wire or at the contacts, this means $G \sim T^{2\alpha_B}$, $G \sim T^{\alpha_B}$ or a combination of both.

IV. PHASE AVERAGING FOR A WIRE WITH FOUR BARRIERS

Finally we mention the case of four barriers. Eq. (3) then leads to

$$\langle |t|^2 \rangle = \frac{\sqrt{T_1 T_2 T_3 T_4}}{\text{agm}\left(\sqrt{V+W}, \sqrt{V-W}\right)}, \qquad (9)$$

where $T_i = |t_i|^2$, $W = 8\,|r_1 r_2 r_3 r_4|$,

$$V = 8 \prod_i \left(1 - \frac{1}{2}T_i\right) + \left(\frac{1}{2} + \sum_i \frac{1-T_i}{T_i^2}\right)\prod_i T_i \quad (10)$$

and the arithmetic-geometric mean $\text{agm}\,(a,b)$ of a and b is defined as the limit of the sequence given by $a_0 = a$, $b_0 = b$, $a_{n+1} = \frac{1}{2}(a_n + b_n)$, $b_{n+1} = \sqrt{a_n b_n}$. For weak barriers, $T_i \approx 1$, this gives again the product law $\langle |t|^2 \rangle \approx T_1 T_2 T_3 T_4$. For strong barriers, $T_i \ll 1$, we get

$$\langle |t|^2 \rangle \approx \frac{1}{2\pi}\sqrt{T_1 T_2 T_3 T_4}\ln\frac{16}{\sqrt{\sum_i T_i^2 + T_1 T_2 T_3 T_4 \sum_i \frac{1}{T_i^2}}}. \qquad (11)$$

The logarithmic factor appearing now causes small deviations from exact power-laws of $G(T)$. However, as for an increased number of barriers higher temperatures are necessary for phase averaging to become a reasonable approximation, this formula might not be as relevant as in the case of three barriers.

V. SUMMARY

We have compared the effects of phase averaging and dephasing in an one-dimensional mesoscopic system. Investigating the scaling behavior of the conductance we have proposed that the two phenomena can be distinguished experimentally by analyzing different kinds and numbers of barriers. This is an important issue for the interpretation of past and future experiments, especially due to the fact that also the contacts to the leads form barriers. In cases where phase averaging dominates, those contact barriers will influence the scaling behavior even if their resistance is lower than the one of barriers within the wire. This might be connected with still unexplained power-law exponents in transport measurements through carbon nanotubes.[6,12,19] Whether phase averaging or dephasing is dominant in these experiments is still an open question and needs further investigations.

[1] P. W. Anderson, D. J. Thouless, E. Abrahams, D. S. Fisher, Phys. Rev. B **22**, 3519 (1980).

[2] Y. Imry and N. S. Shiren, Phys. Rev. B **33**, 7992 (1986).

[3] S. Pilgram, P. Samuelsson, H. Förster, and M. Büttiker, Phys. Rev. Lett. **97**, 066801 (2006)

[4] L. Balents and R. Egger, Phys. Rev. B **64**, 035310 (2001); Phys. Rev. Lett. **85**, 3464 (2000); M. Fabrizio and A. O. Gogolin, Phys. Rev. B **51**, 17827 (1995).

[5] M. Bockrath, D. Cobden, J. Lu, A. G. Rinzler, R. E. Smalley, L. Balents, P. L. McEuen, Nature **397**, 598 (1999); Z. Yao, H. W. Ch. Postma, L. Balents, C. Dekker, Nature **402**, 273 (1999).

[6] H. W. Ch. Postma, T. Teepen, Z. Yao, M. Grifoni, C. Dekker, Science **293**, 76 (2001).

[7] J. König and Y. Gefen, Phys. Rev. Lett. **86**, 3855 (2001).

[8] M. Braun, J. König, and J. Martinek, Phys. Rev. B **70**, 195345 (2004).

[9] R. Landauer, IBM J. Res. Dev. **1**, 223 (1957); M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986); Phys. Rev. **46**, 12485 (1992); Y. Imry, in *Directions in Condensed Matter Physics*, G. Grinstein and G. Mazenko, eds. (World Scientific, Singapore, 1986), p.101.

[10] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995).

[11] For strong hopping barriers, the reflection phases $\delta_i^\pm$ contribute $2ka$ to φ_i, so that the condition of commensurability is to be formulated with $L_i + a$ or $N_i + 1$ instead of L_i or N_i.

[12] T. Enss, V. Meden, S. Andergassen, X. Barnabé-Thériault, W. Metzner, K. Schönhammer, Phys. Rev. B **71**, 155401 (2005).

[13] S. Andergassen, T. Enss, V. Meden, W. Metzner, U. Schollwöck, K. Schönhammer, Phys. Rev. B **70**, 075102 (2004).

[14] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).

[15] D. Yue, L.I. Glazman, and K.A. Matveev, Phys. Rev. B **49**, 1966 (1994).

[16] F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980).

[17] We have neglected the contribution from the scaling of the

bulk density of states since its power-law exponent is of second order in U and therefore usually smaller than the boundary one.

[18] Temperature induced phase averaging becomes effective for $T \gg v_F/L_i$. For $L_i \sim 1\,\mu$m and a semiconductor nanowire, $v_F/L_i \sim 0.5\,$K; for a metallic carbon nanotube, $v_F/L_i \sim 5\,$K.

[19] M. Thorwart, M. Grifoni, G. Cuniberti, H. W. Ch. Postma, C. Dekker, Phys. Rev. Lett. **89**, 196402 (2002); Yu. V. Nazarov and L. I. Glazman, Phys. Rev. Lett. **91**, 126804 (2003); D. G. Polyakov and I. V. Gornyi, Phys. Rev. B **68**, 035421 (2003).