A junction of three quantum wires: restoring time-reversal symmetry by interaction

X. Barnabé-Thériault,* A. Sedeki, V. Meden, and K. Schönhammer
Inst. f. Theoret. Physik, Universität Göttingen, Friedrich-Hund-Platz 1, D-37077 Göttingen, Germany

We investigate transport of correlated fermions through a junction of three one-dimensional quantum wires pierced by a magnetic flux. We determine the flow of the conductance as a function of a low-energy cutoff in the entire parameter space. For attractive interactions and generic flux the fixed point with maximal asymmetry of the conductance is the stable one, as conjectured recently. For repulsive interactions and arbitrary flux we find a line of stable fixed points with vanishing conductance as well as stable fixed points with symmetric conductance $(4/9)(e^2/h)$.

arXiv:cond-mat/0411612v2 [cond-mat.mes-hall] 9 Mar 2005

Electronic transport through quasi one-dimensional (1d) systems is of current experimental and theoretical interest. In one spatial dimension correlations play an important role and the physical properties of interacting fermions show distinctive non-Fermi liquid features. Generically such systems can be described as Tomonaga-Luttinger liquids (TLLs) characterized by a vanishing quasi-particle weight and power-law scaling of correlation functions.[1] For spinless fermions, on which we focus here, the characteristic exponents are expressible in terms of the interaction dependent TLL parameter K. For repulsive interactions $0 < K < 1$, while $K > 1$ in the attractive case. In TLLs inhomogeneities have a dramatic effect as can be inferred from the singular behavior of response functions of homogeneous models.[2, 3]

In an important first step the conductance G through a TLL with a *single impurity* was understood.[4, 5]. For vanishing energy scale s (e.g. temperature) and $0 < K < 1$, G tends towards 0 following a power-law. The low-energy physics is governed by the "decoupled chain" fixed point (FP). The scaling dimension of a hopping term connecting two open ends is $1/K$ and leads to $G \propto s^{2(1/K-1)}$. One can understand this behavior in a simple picture. Due to the interaction the self-energy develops long range oscillatory behavior and the scattering off the resulting effective impurity potential leads to the power-law suppression of G.[6, 7, 8] For $K > 1$, the conductance approaches the impurity free limit. Close to the "perfect chain" FP an impurity has scaling dimension K and the correction to the impurity free conductance scales as $s^{2(K-1)}$. In this case the effective impurity potential leads to a resonance at the chemical potential.

Recently junctions of several quasi 1d quantum wires were realized experimentally with single-walled carbon nanotubes.[9, 10] They might form the basis of electronic devices. Already the physics of the three wire junction (Y-junction) is considerably richer than the one of a single impurity. Taking into account correlations transport through such systems was investigated theoretically in Refs. 11 and 12. These studies posed a number of interesting questions. In Ref. 12 a symmetric triangular Y-junction pierced by a magnetic flux ϕ (measured in units of the flux quantum hc/e) was studied. In this geometry time reversal symmetry is broken. In the non-interacting case this generically leads to an asymmetry of the conductance from wire ν to wire ν' and vice versa: $G_{\nu,\nu'} \neq G_{\nu',\nu}$. For $K > 1$ at flux $\phi = \pm\pi/2$ one FP was found applying an exact method adopted from boundary conformal field theory. The FP corresponds to the case of *maximal asymmetry* of $G_{\nu,\nu'}$. We here consider TLL wires connected to semi-infinite Fermi liquid (FL) leads with TLL-FL contacts that are modeled to be free of fermion backscattering. The results of Ref. 12 obtained for semi-infinite TLLs can be extended to our modeling. Then maximal asymmetry is given by $G_{\nu,\nu'} = 1(0)$ and $G_{\nu',\nu} = 0(1)$ in units of the conductance quantum e^2/h.[13] The scaling dimension of the most relevant operator at this FP is $\Delta = 4K/(3 + K^2)$ and the correction to the FP conductance scales as $s^{2(\Delta-1)}$.[12] For $1 < K < 3$, $2(\Delta - 1) > 0$ and the "maximal asymmetry" FP is attractive. This implies that independently of the junction parameters at low energy scales the conductance is 1 from ν to ν' and 0 from ν' to ν or vice versa. It was *conjectured* that for $1 < K < 3$ and *all* ϕ, except $|\phi|/\pi$ being integer, this FP is the only stable one.

We investigate the same physical problem considering *arbitrary fluxes* and attractive as well as *repulsive interactions*. An approximation scheme based on the functional renormalization group (fRG) method is used. It was earlier applied successfully to transport problems in inhomogeneous TLLs. By comparison with numerical results and exact scaling relations this scheme was shown to be reliable for $1/2 \leq K \leq 3/2$. In particular, the scaling dimensions of the two FPs of the single impurity problem come out correctly to leading order in the interaction U.[7, 8] For the Y-junction we here confirm the conjectured stability of the "maximal asymmetry" FP for *all* ϕ, with $|\phi|/\pi$ not integer, and reproduce the scaling dimension Δ to leading order in U. For $U > 0$ and arbitrary ϕ a line of stable FPs with $G = 0$ is identified. In cases with symmetric conductance we suppress the indices on G. We surprisingly find additional "perfect junction" FPs with *symmetric* conductance $G = 4/9$ that for $U > 0$ are *stable* and have a scaling dimension not discussed before. In a Y-junction of non-interacting wires without flux $G = 4/9$ is the value maximally allowed by symmetry.[11] Although time reversal symmetry is explicitly broken, for systems that flow towards the "perfect junction" FPs the *electron correlations* cause the conductance – an observable that is commonly believed to indicate this breaking of symmetry – to behave as if the symmetry is restored. In a certain sense (see below) this also holds for systems that flow towards the line of $G = 0$ FPs and thus for all parameters.

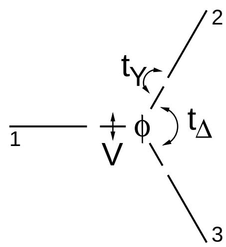

FIG. 1: Symmetric junction of three quantum wires.

Our wire Hamiltonian is

$$H_\nu = -\sum_{j=1}^{\infty}\left(c^\dagger_{j,\nu}c_{j+1,\nu} + \text{h.c.}\right) + \sum_{j=1}^{N-1} U_j \left(n_{j,\nu} - \frac{1}{2}\right)\left(n_{j+1,\nu} - \frac{1}{2}\right) \qquad (1)$$

in standard second-quantized notation. The different wires are indicated by an index $\nu = 1, 2, 3$. The amplitude of the nearest neighbor hopping is set to 1. The nearest neighbor interaction U_j is allowed to depend on the position. It is set to zero for $j > N$, i.e. the wires of length N are connected to non-interacting leads. Close to the contacts the interaction is switched off (spatially) smoothly to avoid any fermion backscattering.[8] The bulk value of the interaction is denoted by U. We here consider the half-filled band case. To prevent depletion of the interacting region we shifted the operator $n_{j,\nu}$ by the average density 1/2. The model with interaction U across all bonds (not only the ones within $[1, N]$) shows TLL behavior for $|U| < 2$ with[14]

$$K = \left[\frac{2}{\pi}\arccos\left(-\frac{U}{2}\right)\right]^{-1} . \qquad (2)$$

The Y-junction sketched in Fig. 1 is described by

$$H_\text{Y} = -t_\text{Y}\sum_{\nu=1}^{3}\left(c^\dagger_{1,\nu}c_{0,\nu} + \text{h.c.}\right) + V\sum_{\nu=1}^{3} n_{0,\nu} - t_\triangle \sum_{\nu=1}^{3}\left(e^{i\phi/3}c^\dagger_{0,\nu}c_{0,\nu+1} + \text{h.c.}\right) , \qquad (3)$$

where we identify the wire indices 4 and 1.

Our starting point to calculate the linear response conductance of $\sum_{\nu=1}^{3} H_\nu + H_\text{Y}$ is an exact hierarchy of differential flow equations for the self-energy Σ^Λ and higher order vertex functions (in the presence of the interaction, the leads, and the junction), with an infrared energy cutoff Λ as the flow parameter. It is derived using the fRG. The hierarchy is truncated by neglecting n-particle vertices with $n > 2$, and the 2-particle vertex is parametrized by a renormalized nearest neighbor interaction U^Λ. This implies that terms of order U^2 are only partly taken into account and that Σ^Λ is frequency independent. The important spatial dependence of Σ^Λ, is however fully kept. The details of this procedure are given in Refs. 7 and 8. At the end of the flow (at $\Lambda = 0$), the self-energy can be regarded as an effective, N-dependent impurity potential Σ with non-vanishing matrix elements $\Sigma_{j,j}$ and $\Sigma_{j,j+1}$, where j is restricted to the interacting region. Due to the symmetry of the junction the matrix elements are the same for the three TLL wires. We here focus on the zero temperature case for which the flow equations can numerically be solved for up to $N = 10^7$ lattice sites.[7, 8] Generically $\Sigma_{j,j}$ and $\Sigma_{j,j+1}$ oscillate around an average value with an amplitude that decays slowly with increasing distance from the junction.

The conductance $G_{\nu,\nu'} = |t_{\nu,\nu'}|^2$ can be calculated from the effective transmission $t_{\nu,\nu'}$ (at the chemical potential $\mu = 0$), which in turn can be expressed in terms of real space matrix elements of one-particle Green functions of the system. Using single particle scattering theory,[8] the conductance can be written as

$$G_{\nu,\nu'} = \frac{4\left(\operatorname{Im} g\right)^2 \left|e^{-i\phi} - g\right|^2}{\left|g^3 - 3g + 2\cos\phi\right|^2} , \qquad (4)$$

with $g = (-V - t_Y^2\,\mathcal{G}_{1,1})/|t_\triangle|$. The Green function $\mathcal{G}$ is obtained by considering Σ as an effective potential for a single wire setting $t_Y = 0$ and $\mathcal{G}_{1,1}$ denotes its diagonal matrix element taken at site $j = 1$. It is evaluated at energy $\varepsilon + i0$ with $\varepsilon \to 0$. Eq. (4) holds if ν, ν' are in cyclic order. $G_{\nu',\nu}$ follows by replacing $\phi \to -\phi$. For symmetry reasons we only have to consider $0 \le \phi \le \pi/2$. In particular Eq. (4) can be applied for $U = 0$ and shows that the conductance can be parameterized by the flux and a *single complex* parameter g. Via the flow of the self-energy $\mathcal{G}_{1,1}$ for $U \neq 0$ develops an additional dependence on $(t_Y, t_\triangle, V)$, U, and most importantly on N. The energy scale $\delta_N = \pi v_F/N$ (with the Fermi velocity v_F) is a natural infrared cutoff of our problem.[8] To obtain a comprehensive picture of the low-energy physics we investigate the flow of g as a function of δ_N and use Eq. (4) to calculate $G_{\nu,\nu'}$ for a given g. In Fig. 2 the flow of g is shown for different ϕ (and $U = -1$). Each line is obtained for a fixed set of junction parameters $(t_Y, t_\triangle, V)$ with N as a variable. As $\operatorname{Im} g$ has the opposite sign of $\operatorname{Im} \mathcal{G}_{1,1} < 0$ it is restricted to positive values.

Eq. (4) allows for *three* special conductance situations which turn out to be the FPs of the flow. On the real axis (green in Fig. 2), the conductance G *vanishes* except at special, ϕ-dependent points. They are given by the crossings of the real axis with the set $\mathcal{C}(\phi)$ (red in Fig. 2), on which the reflection $R = 1 - |t_{\nu,\nu'}|^2 - |t_{\nu',\nu}|^2$ takes a local minimum. For $\phi = 0$, $\mathcal{C}(0)$ is given by the circle

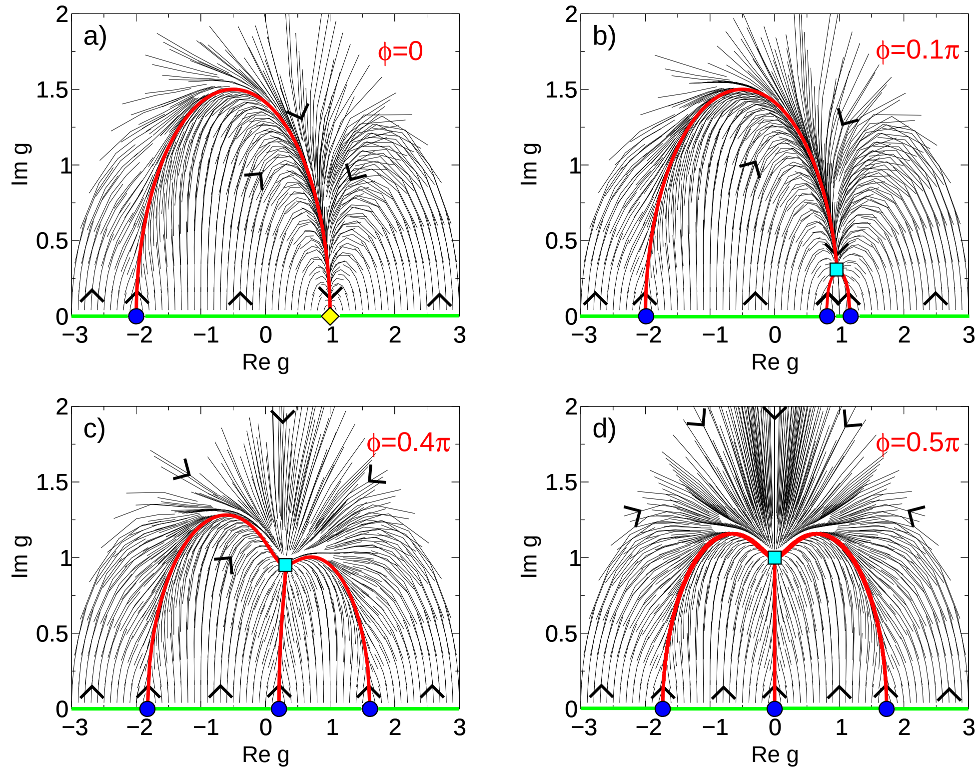

FIG. 2: (color online) Flow of g for different ϕ. Arrows indicate the direction for $U < 0$. For $U > 0$ it is reversed. For details see the text.

$(\mathrm{Re}\, g + 1/2)^2 + (\mathrm{Im}\, g)^2 = (3/2)^2$. For $\phi > 0$, $\mathcal{C}(\phi)$ has a more complex analytical form not presented here. At the crossings the conductance is *symmetric* with $G = 4/9$. They are located at $g = -2\cos(\phi/3)$, $2\cos([\pi \pm \phi]/3)$ and in Fig. 2 are indicated as blue circles and a yellow diamond. As a peculiarity of $\phi = 0$ on $\mathcal{C}(0)$ one finds $G = 4/9$ for all $\mathrm{Im}\, g$, not only $\mathrm{Im}\, g = 0$. A situation with $G = 0$ is also reached for $|g| \to \infty$. For $0 < \phi \leq 1/2$ and $g = e^{i\phi}$ Eq. (4) yields $G_{\nu,\nu'} = 1$ and $G_{\nu',\nu} = 0$. At this point, indicated by cyan squares in Fig. 2, *maximal asymmetry* of the conductance is achieved.

We next discuss the FP scenarios depicted in Figs. 2 a)-d). For $U \neq 0$ the general form of the flow diagrams is *independent* of the absolute value and sign of U. In Figs. 2 a)-d) results for $U = -1$ are shown. At $\phi = 0$ we find two "perfect junction" FPs with $G = 4/9$ and a line of "decoupled chain" FPs with $G = 0$ (green line). For all $U < 0$ the "perfect junction" FP at $g = 1$ (yellow diamond) is the only stable FP. All trajectories are attracted towards $\mathcal{C}(0)$ and reach this FP following $\mathcal{C}(0)$. For all $U > 0$ it turns unstable and the line of "decoupled chain" FPs is stable. In addition the "perfect junction" FP at $g = -2$ (blue circle) is stable. The *basin of attraction* of $g = -2$ is given by $\mathcal{C}(0)$. Increasing ϕ from 0 the "perfect junction" FP at $g = 1$ splits up into *three* FPs – the two "perfect junction" FPs at $g = 2\cos([\pi \pm \phi]/3)$ and the "maximal asymmetry" FP at $g = e^{i\phi}$. For all $U < 0$ the latter is the *only* stable FP but becomes unstable for $U > 0$. A third "perfect junction" FP is located at $g = -2\cos(\phi/3)$. For all $U > 0$ each of the three "perfect junction" FPs has a *basin of attraction* given by one of the three parts of $\mathcal{C}(\phi)$ which are separated by the "maximal asymmetry" FP. In addition for $U > 0$ the line of "decoupled chain" FPs is stable. This scenario holds up to $\phi = \pi/2$, at which the "perfect junction" FPs are at $g = \pm\sqrt{3}$, $g = 0$, and the "maximal asymmetry" FP is at $g = i$. For $\phi = \pi/2$ and $U > 0$ there is a single trajectory that runs along the imaginary axis to infinity (leading to $G = 0$) and does not bend back towards the real axis as all other trajectories at $U > 0$ do. In the mapping of the complex plane onto the Riemann sphere the $g = \infty$ FP (north pole) is part of the projected line of "decoupled chain" FPs and shows the same stability properties and scaling dimension.

To obtain the scaling dimensions of the FPs we generically analyze the scaling of $G_{\nu,\nu'}$ as a function of δ_N with respect to the FP conductance which is 1, 4/9, or 0 depending on the FP studied. For sufficiently large N (in some cases up to 10^5 sites are required) we find power-law scaling close to all FPs. In cases were the FPs are not stable this scaling does not present the asymptotic behavior, but exponents smaller than 0 can still be read off at intermediate N. Approaching the "perfect junction" FPs along $\mathcal{C}(\phi)$ for $\phi > 0$ Eq. (4) yields

$G_{\nu,\nu'} - 4/9 \propto \mathrm{Im}\, g$, i.e. $G_{\nu,\nu'} - 4/9$ and $\mathrm{Im}\, g$ scale with the same exponent. The scaling dimensions of the "perfect junction" FPs at $\phi = 0$ cannot be read off from the conductance as $G = 4/9$ on $\mathcal{C}(0)$ and we use the scaling of $\mathrm{Im}\, g$.

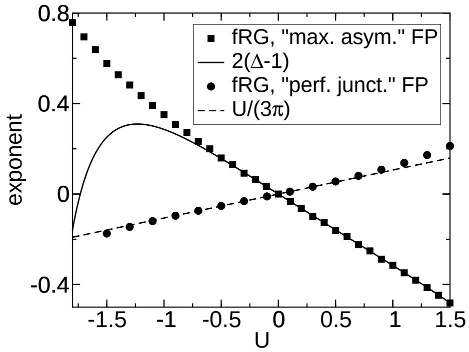

FIG. 3: Scaling exponents close to FPs.

The scaling exponent of the "maximal asymmetry" FP is *independent* of the flux $\phi > 0$ and the direction from which it is approached ($U < 0$) or in which it is left ($U > 0$). Its U-dependence is shown in Fig. 3 (squares) and agrees to leading order in U with the prediction $2(\Delta - 1)$ (solid line),[12] with $\Delta = 4K/(3 + K^2)$ and K given in Eq. (2). In our scheme terms of order U^2 are only partly included and we cannot expect agreement to higher order. The non-monotonic behavior of $2(\Delta - 1)$ is not reproduced and in our approximation the "maximal asymmetry" FP stays attractive for all $U < 0$. For small to intermediate $|U|$ with TLL parameters $1/2 \leq K \leq 3/2$ we confirm the conjecture of Ref. 12.

For the line of "decoupled chain" FPs we find for all ϕ the same scaling exponent β_s, as found for the *single impurity* problem close to the respective "decoupled chain" FP, applying the fRG. Its dependence on U is shown in Ref. 8 and agrees to leading order with $2(1/K - 1)$.

As discussed above the $\phi = 0$ "perfect junction" FP at $g = 1$ [yellow diamond in Fig. 2 a)] has properties different from those of the other "perfect junction" FPs (blue circles in Fig. 2). The scaling exponent γ of the latter FPs is independent of ϕ and shown in Fig. 3 (circles). To leading order it is given by $\gamma \approx U/(3\pi)$ (dashed line). This form does *not* coincide with the expansion of any of the above K-dependent exponents [after using $K \approx 1 - U/\pi$; see Eq. (2)]. We thus find a *new* scaling dimension. Since higher order terms are only partly included we cannot determine the functional dependence of γ on K. To derive such an expression presents a challenge for methods that do not require approximations in the strength of the interaction. For the $\phi = 0$ "perfect junction" FP at $g = 1$ the scaling exponent (of $\mathrm{Im}\, g$) is, up to our numerical accuracy, equal to $-\gamma$. For junction parameters that initially do not fall onto $\mathcal{C}(0)$, but are close to it, one can read off an exponent from the conductance that describes how $\mathcal{C}(0)$, with $G = 4/9$, is approached ($U < 0$) or left ($U > 0$). It is equal to the fRG exponent β_w of the "perfect chain" FP in the *single impurity* problem. Its U-dependence is presented in Ref. 8 and agrees with $2(K - 1)$ to leading order.

The appearance of stable FPs with symmetric conductance $G = 4/9$ at $U > 0$ is a surprising result. Even though time reversal symmetry is explicitly broken at $\phi > 0$, due to correlations the conductance of systems with parameters on $\mathcal{C}(\phi)$ behaves as if time reversal symmetry is restored. It is remarkable that close to the line of "decoupled chain" FPs the relative difference $|G_{\nu,\nu'} - G_{\nu',\nu}|/(G_{\nu,\nu'} + G_{\nu',\nu})$ scales as $\delta_N^{\beta_s/2}$ and thus vanishes if $U > 0$. This implies that $G_{\nu,\nu'}$ and $G_{\nu',\nu}$ *become equal* faster than they go to zero. In that sense also on this line of FPs and thus for *all* junction parameters time reversal symmetry is restored if $U > 0$.

Using the fRG we determined the complete renormalization group flow for a TLL Y-junction pierced by a magnetic flux. Besides uncovering a new type of low-energy physics we demonstrated the power of our approximation scheme. Usually field theoretical models are used to investigate the transport properties of inhomogeneous TLL applying methods that are specific to such models. This way *exact* results for either fairly simple geometries (single impurity[5]) or restricted parameter regimes[12] were obtained. Our method can be applied to *microscopic models* with *arbitrary junction parameters* and provides results for the conductance that are accurate for small to intermediate $|U|$. It can also be used to study transport on *intermediate* and *large* energy scales.[8] Furthermore, the technique can be applied to investigate the transport through more complex networks of TLLs that might become important in future nano-electronic applications.

We thank C. Chamon, S. Andergassen, T. Enss, W. Metzner, and Th. Pruschke for discussions. V.M. and K.S. are grateful to the Deutsche Forschungsgemeinschaft (SFB 602) for support.

* Xavier B.-T. passed away in a tragic traffic accident on August 15, 2004.

[1] For a review see K. Schönhammer in *Interacting Electrons in Low Dimensions,* Ed.: D. Baeriswyl, Kluwer Academic Publishers (2004); cond-mat/0305035.
[2] A. Luther and I. Peschel, Phys. Rev. B **9**, 2911 (1974).
[3] D. Mattis, J. Math. Phys. **15**, 609 (1974).
[4] C. Kane and M. Fisher, Phys. Rev. Lett. **68**, 1220 (1992); Phys. Rev. B **46**, 15233 (1992).
[5] P. Fendley, A. Ludwig, and H. Saleur, Phys. Rev. Lett. **74**, 3005 (1995).
[6] D. Yue, L. Glazman, and K. Matveev, Phys. Rev. B **49**, 1966 (1994).
[7] V. Meden *et al.,* Phys. Rev. B **65**, 045318 (2002); J. of Low Temp. Physics **126**, 1147 (2002); S. Andergassen *et al.,* Phys. Rev. B **70**, 075102 (2004).
[8] T. Enss *et al.,* to appear in Phys. Rev. B (2005).

[9] M. Fuhrer *et al.,* Science **288**, 494 (2000).
[10] M. Terrones *et al.,* Phys. Rev. Lett. **89**, 075505 (2002).
[11] C. Nayak *et al.,* Phys. Rev. B **59**, 15694 (1999); S. Lal, S. Rao, and D. Sen, *ibid.* **66**, 165327 (2002); S. Chen, B. Trauzettel, and R. Egger, Phys. Rev. Lett. **89**, 226404 (2002).
[12] C. Chamon, M. Oshikawa, and I. Affleck, Phys. Rev. Lett. **91**, 206403 (2003).
[13] M. Oshikawa, C. Chamon, and I. Affleck, in preparation.
[14] F.D.M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980).