

Role of a spin-flip scatterer in a magnetized Luttinger liquid

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We study the spin-dependent scattering of charge carriers in a magnetized one dimensional Luttinger liquid from a localized non-homogeneous magnetic field, which might be brought about by the stray field of magnetic tip near a uniform liquid, or by a transverse domain wall (DW) between two oppositely magnetized liquids. From a renormalization group treatment of the electron interactions we deduce scaling equations for the transmission and reflection amplitudes as the bandwidth is progressively reduced to an energy scale set by the temperature. The repulsive interactions dictate two possible zero temperature insulator fixed points: one in which electrons are reflected in the same spin channel and another where the electron spin is reversed upon reflection. In the latter case, a finite spin current emerges in the absence of a charge current at zero temperature and the Friedel oscillations form a transverse spiraling spin density. Adding a purely potential scattering term has no effect on the fixed points of a uniformly magnetized liquid. For a DW we find that the introduction of potential scattering stabilizes the spin-flip insulator phase even if the single-particle spin-flip scattering produced by the DW is arbitrarily weak. The potential can be induced externally, e.g. by a local gate voltage or a constriction, providing a means for controlling the transport properties of the wire.

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I. INTRODUCTION

Transport properties of magnetic nanowires are currently in the focus of intense research in view of possible applications in magneto-electronic devices^{1,2,3,4,5}. Particularly interesting are magnetic nanowires with a localized topological magnetic disorder which acts as a spin-dependent scatterer of charge carriers. Examples of this situations are magnetized wires with domain walls or magnetic microvortices that can be well controlled externally by a magnetic field or by an electric current.

Here, we address the problem of scattering of interacting electrons from a single defect in a magnetic nanowire, assuming that the nanowire is thin enough so that the electron energy spectrum is one-dimensional. A similar problem in the nonmagnetic case attracted a lot of attention in the past^{6,7} because of the key role of electron-electron interactions leading to substantial renormalization of the scattering amplitudes.

Recently⁸, we treated the problem of the spin dependent scattering in a short transverse domain wall (DW) separating two oppositely magnetized regions in a 1D wire. In particular, we considered the case of a DW whose extension is comparable to the carriers' de Broglie wavelength, in which case the influence of scattering and interactions is particularly strong. On the other hand, this limitation implies severe demands on an experimental realization restricting possible systems to magnetic semiconductor nanowires, such as (Ga,Mn)As⁹. A further complication is brought about by magnetic impurities in the semiconductor that cause scattering in the

region of the domain wall¹⁰. If the DW is pinned by a constriction, the constriction itself behaves as a pure potential scatterer.

There is yet another possibility for inducing localized magnetic imperfections in a wire: the stray magnetic field of a nearby tip of a spin-polarized scanning tunneling microscope (STM) in the T-shape geometry¹¹ illustrated in Fig.1. In this case, the wire itself may be magnetically homogeneous and free of impurities.

For magnetic nanowires the scattering involves at least two channels and different components of the transmission amplitudes may well be renormalized in different ways. As shown below, this results in some interesting effects such as a spin-flip insulator state. The possibility of these effects has been demonstrated recently for scattering from a magnetic transverse domain wall^{8,12}. In the case of a homogeneous magnetization of the wire, possible types of defects can be classified as purely potential defects (i.e., a local perturbation that affects equally the spin up and down electrons), local variation of the magnitude of the magnetization, and a local variation of the magnetization direction. The first two cases are straightforwardly treated since they can be analyzed in terms of potential scattering in each of the spin channels. The problem tackled here is the case of a local variation of the magnetization direction, when both spin-flip and non-spin-flip scattering amplitudes are renormalized, by electron interactions, in different ways.

Technically, we need to address the scattering of electrons from a localized transverse magnetic field. The single-particle scattering (either pure potential or spin

dependent) can be treated exactly. We then address the role of electron interactions perturbatively, to first order, resulting in the well-known logarithmic divergences. The divergent terms are then circumvented by a poor man's scaling approach that yields a set of renormalization equations for the scattering amplitudes.

The two problems have similar fixed point solutions at zero temperature ($T = 0$). Considering repulsive spin dependent interactions, two types of insulator may arise, depending on the interaction parameters: the electrons may be 100% reflected with or without spin reversal. In the former case there is a spin current without charge current and the spin current exerts a torque on the magnetic tip. For the case of a DW we find the counter-intuitive effect that a non-spin-flip insulator phase is obtained if the DW's transverse field is weak. But by adding a purely potential scattering term, one is able to drive the DW to the spin-flip insulator phase. The potential itself can be externally imparted, e.g. by a constriction or by a local gate voltage, much in the same way as demonstrated by recent experiments¹³.

Section II introduces the electronic Hamiltonian for the problems depicted in Figure 1 and the single-particle solutions. Section III gives the renormalization group equations for the scattering amplitudes in each problem, which are a consequence of the electron interactions. Section IV contains a discussion and summary.

II. MODEL HAMILTONIAN

We consider a ferromagnetic metallic wire close to a magnetic tip as shown in Figure 1(a). The latter produces an effective magnetic field which acts as a spin-flip scatterer in a localized region of the wire. The wire defines the easy (\hat{z}) axis with uniform magnetization $\mathbf{M} = M\hat{z}$. The effective magnetic field $B_{\perp}(z)\hat{x} + B_{\parallel}(z)\hat{z}$ due to the influence of the tip which is placed nearly perpendicular to the wire affects a small region near $z = 0$. The conduction electron spin in the wire is Zeeman coupled to this magnetic field. We treat the conduction electrons as one-dimensional and write the single particle Hamiltonian as:

$$\hat{H}_0 = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + \hbar V \delta(z) - JM_0 \hat{\sigma}_z - \hbar \lambda \delta(z) \hat{\sigma}_x - \hbar \lambda' \delta(z) \hat{\sigma}_z. \quad (1)$$

The terms $\hbar \lambda^{(\prime)} \delta(z) \hat{\sigma}_x$, describe spin scattering produced by the the magnetic tip:

$$\begin{aligned} \hbar \lambda &= \mu \int_{-\infty}^{\infty} B_{\perp}(z) dz, \\ \hbar \lambda' &= \mu \int_{-\infty}^{\infty} B_{\parallel}(z) dz, \end{aligned}$$

and μ is the electron's magnetic moment. We also allow for a purely potential scattering term, V , that may be present.

In the case where the wire has two opposite ferromagnetic domains separated by a thin transverse DW, as shown in Figure 1(b), the single particle Hamiltonian is:

$$\hat{H}_{DW} = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + \hbar V \delta(z) - JM(z) \hat{\sigma}_z - \hbar \bar{\lambda} \delta(z) \hat{\sigma}_x. \quad (2)$$

where the term $\hbar \bar{\lambda} \delta(z) \hat{\sigma}_x$ now produces spin-flip scattering due to the x component of the magnetization in the DW:

$$\hbar \bar{\lambda} = -J \int_{-\infty}^{\infty} M_x(z) dz$$

and the longitudinal magnetization $M(z) = -M_0$ for $z < 0$ and $M(z) = M_0$ for $z > 0$. In this case the longitudinal magnetization produces a Zeeman potential step which scatters electrons more strongly as compared to the magnetic tip problem above.

In both models, a single spin-majority electron may be transmitted either preserving or reversing its spin with amplitudes t_{\uparrow} or t'_{\uparrow} , respectively. A spin minority electron has transmission amplitudes t_{\downarrow} or t'_{\downarrow} . The reflection amplitudes in the same (opposite) spin channel are denoted by r_{\uparrow} (r'_{\uparrow}) for spin majority and spin minority electrons, respectively.

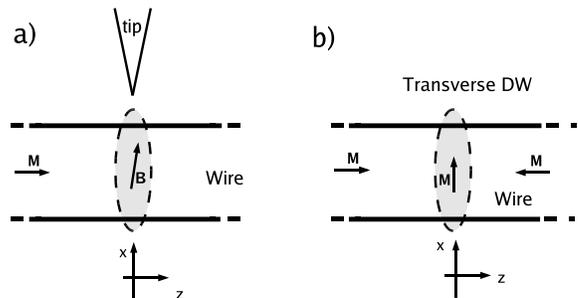


FIG. 1: (a) Magnetic tip produces a nearly transverse magnetic field in a localized region of a uniformly magnetized metallic wire; (b) a transverse DW in the wire also acts as a transverse field.

We wish to consider the effect of electron interactions on the scattering amplitudes. The interactions can be described by the g -ology model¹⁴:

$$\begin{aligned} \hat{H}_{int} &= g_{1,\alpha,\beta} \int \frac{dk_1 dq}{(2\pi)^2} \hat{a}_{k_1,\alpha}^\dagger \hat{b}_{k_2,\beta}^\dagger \hat{a}_{k_2+q,\beta} \hat{b}_{k_1-q,\alpha} \\ &+ g_{2,\alpha,\beta} \int \frac{dk_1 dq}{(2\pi)^2} \hat{a}_{k_1,\alpha}^\dagger \hat{b}_{k_2,\beta}^\dagger \hat{b}_{k_2+q,\beta} \hat{a}_{k_1-q,\alpha}. \end{aligned} \quad (3)$$

The couplings g_1 and g_2 describe back and forward scattering processes between opposite moving electrons, respectively, and are positive if the interactions are repulsive. Because the Fermi momentum depends on spin, we allow for the dependence of g on the spins of the interacting particles. We therefore distinguish between $g_{1\uparrow}$, $g_{1\downarrow}$

$g_{1\perp}$ and $g_{2\uparrow}$, $g_{2\downarrow}$, $g_{2\perp}$. The g_2 processes imply zero momentum transfer whereas the g_1 processes involve a $2k_F$ momentum transfer. We therefore expect g_2 larger than g_1 for a finite spatial range of the repulsive interactions.

Our treatment of the interactions follows the same method invented in Ref⁷ and developed in Ref⁸: the corrections to the scattering amplitudes are calculated to first order in the perturbation \hat{H}_{int} , leading to logarithmic divergent terms if the electron is near the Fermi level. Using a poor man's scaling procedure, the divergent terms lead to a renormalization of the scattering amplitudes as the bandwidth, D , is progressively reduced from its initial value, D_0 , to $D = T$. The fixed points are attained at temperature $T = 0$.

III. SCALING EQUATIONS

We write down the renormalization group (RG) equations for the scattering amplitudes, in each problem, using the variable $\xi = \log(D/D_0)$ which will be integrated from 0 to $\log(T/D_0)$, corresponding to the fact that the bandwidth is progressively reduced from D_0 to T . We also define:

$$g_\sigma = \frac{g_{2\sigma} - g_{1\sigma}}{4\hbar v_\sigma}, \quad g_\perp = \frac{g_{2\perp}}{2\hbar(v_\downarrow + v_\uparrow)}. \quad (4)$$

where v_\uparrow (v_\downarrow) denotes the Fermi velocity of spin majority (minority) particles.

The RG equations result from a first order perturbative treatment of the interactions. The total scattering (transmission or reflection) amplitude is the sum over all virtual scattering processes involving the electron-electron interaction once⁸. The latter occurs in the form of Bragg back-scattering caused by the $2k_F$ Hartree-Fock potential caused by the Fermi sea Friedel oscillations. So, the simplest process is just one Bragg reflection produced by the Friedel oscillation. The other possible process involve three virtual scattering events: the electron first collides with the barrier, then with the Friedel oscillation and finally with the barrier again.

A. Thin DW

A detailed derivation of the RG equations for this case has been given elsewhere⁸ and we merely reproduce them here:

$$\begin{aligned} \frac{dt_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 t_\sigma + r_\sigma^* r'_\sigma t'_\sigma) \\ &+ g_{-\sigma} (|r_{-\sigma}|^2 t_\sigma + r_{-\sigma}^* r'_\sigma t'_{-\sigma}) \\ &+ g_\perp (2r_{-\sigma}^* r'_\sigma t_\sigma + r_{-\sigma}^* r_{-\sigma} t'_\sigma + r_{-\sigma}^* r_\sigma t'_{-\sigma}), \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{dt'_\sigma}{d\xi} &= 2g_\sigma |r_\sigma|^2 t'_\sigma + 2g_{-\sigma} r_{-\sigma}^* r'_{-\sigma} t_\sigma \\ &+ 2g_\perp (r_{-\sigma}^* r_\sigma t_\sigma + r_{-\sigma}^* r'_{-\sigma} t'_\sigma), \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{dr_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 r_\sigma + r_\sigma^* t_\sigma'^2 - r_\sigma) \\ &+ g_{-\sigma} (r_{-\sigma}^* t_\sigma t_{-\sigma} + r_{-\sigma}^* r'_{-\sigma} r'_\sigma) \\ &+ 2g_\perp (r_{-\sigma}^* r'_{-\sigma} r_\sigma + r_{-\sigma}^* t_{-\sigma} t'_\sigma), \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{dr'_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 r'_\sigma + r_\sigma^* t'_\sigma t_\sigma) \\ &+ g_{-\sigma} (r_{-\sigma}^* t_\sigma t'_{-\sigma} + r_{-\sigma}^* r_{-\sigma} r'_\sigma) \\ &+ g_\perp (r_{-\sigma}^* r_{-\sigma} r_\sigma + r_{-\sigma}^* r_\sigma'^2 + r_{-\sigma}^* t'_{-\sigma} t'_\sigma \\ &+ r_{-\sigma}^* t_\sigma'^2 - r'_\sigma). \end{aligned} \quad (8)$$

The Wronskian theorem^{8,15} for the scattering problem establishes the following relations between the scattering amplitudes:

$$|r_\sigma|^2 + |t'_\sigma|^2 + \frac{v_{-\sigma}}{v_\sigma} (|r'_\sigma|^2 + |t_\sigma|^2) = 1, \quad (9)$$

$$r_\sigma^* r'_{-\sigma} + t'_\sigma t_{-\sigma} + \frac{v_{-\sigma}}{v_\sigma} (r_{-\sigma}^* r_{-\sigma} + t_\sigma^* t'_{-\sigma}) = 0, \quad (10)$$

$$r_\sigma^* t_{-\sigma} + t'_\sigma r'_{-\sigma} + \frac{v_{-\sigma}}{v_\sigma} (r_{-\sigma}^* t'_{-\sigma} + t_\sigma^* r_{-\sigma}) = 0, \quad (11)$$

$$\text{Re}[t_\sigma^* r_\sigma] + \frac{v_{-\sigma}}{v_\sigma} \text{Re}[r_{-\sigma}^* t_{-\sigma}] = 0, \quad (12)$$

$$v_\sigma t_{-\sigma} = v_{-\sigma} t_\sigma, \quad v_\sigma r'_{-\sigma} = v_{-\sigma} r'_\sigma. \quad (13)$$

The initial non-interacting values ($\xi = 0$) of the scattering amplitudes near the Fermi level are obtained from the Hamiltonian (2):

$$t_\sigma = \frac{2v_\sigma(v_\sigma + v_{-\sigma} + 2iV)}{(v_\sigma + v_{-\sigma} + 2iV)^2 + 4\bar{\lambda}^2} = r_\sigma + 1, \quad (14)$$

$$t'_\sigma = \frac{4i\bar{\lambda}v_\sigma}{(v_\sigma + v_{-\sigma} + 2iV)^2 + 4\bar{\lambda}^2} = r'_\sigma. \quad (15)$$

In the absence⁸ of potential scattering ($V = 0$), $t_\sigma(\xi)$ and $r_\sigma(\xi)$ are real while $t'_\sigma(\xi)$ and $r'_\sigma(\xi)$ are pure imaginary. In this case, two types of insulator fixed points were found⁸: the ordinary insulator with $t_\sigma = t'_\sigma = r'_\sigma = 0$, $|r_\sigma| = 1$, which is attained if $g_\perp < (g_\uparrow + g_\downarrow)/2$; the "spin-flip reflector" with $t_\sigma = t'_\sigma = r_\sigma = 0$, $|r'_\sigma| = \sqrt{v_\uparrow/v_\downarrow}$, which is attained for $g_\perp > (g_\uparrow + g_\downarrow)/2$ and not small $\bar{\lambda}/v_\uparrow$.

Our aim here is to analyze the effect of the purely potential term V . For small $\bar{\lambda}/v_\uparrow$ and $V = 0$, the DW flows to the zero temperature fixed point with $t = t' = r' = 0$ and $r_\uparrow = -r_\downarrow = 1$. The particles are reflected in the same spin channel because the spin-flip term $\bar{\lambda}/v_\uparrow$ is not strong enough and the Zeeman potential step $-JM(z)\hat{\sigma}_z$ in (2) is dominant. For finite potential scattering, V , all the amplitudes become complex. We may study the RG flow near this fixed point by linearizing equation (8) for $\sigma = \uparrow$ in the small quantity r'_\uparrow :

$$\frac{dr'_\uparrow}{d\xi} \approx (g_\uparrow + g_\downarrow - g_\perp)r'_\uparrow + g_\perp r_\uparrow r_\downarrow r'^*_{\uparrow}, \quad (16)$$

which, for $r_\uparrow r_\downarrow = -1$, gives the scaling of the real and imaginary parts as

$$\text{Re}[r'_\uparrow] \propto e^{(g_\uparrow + g_\downarrow - 2g_\perp)\xi}, \quad \text{Im}[r'_\uparrow] \propto e^{(g_\uparrow + g_\downarrow)\xi}, \quad (17)$$

implying that $\text{Im}[r'_\uparrow]$ scales to zero as $T \rightarrow 0$ (or $\xi \rightarrow -\infty$), while $\text{Re}[r'_\uparrow]$ is relevant for $g_\perp > (g_\uparrow + g_\downarrow)/2$ and irrelevant if $g_\perp < (g_\uparrow + g_\downarrow)/2$. Since the real part of r'_\uparrow is due to the finiteness of V , we see that the potential scattering will enhance the spin-flip reflection processes if $g_\perp > (g_\uparrow + g_\downarrow)/2$. For such interactions, the $r'_\sigma = 0$ fixed point is in fact *unstable to any small V* and the system will flow to the other fixed point with $r_\sigma = 0$ and $|r'_\uparrow| = \sqrt{v_\uparrow/v_\downarrow}$. This is shown in Figure 2.

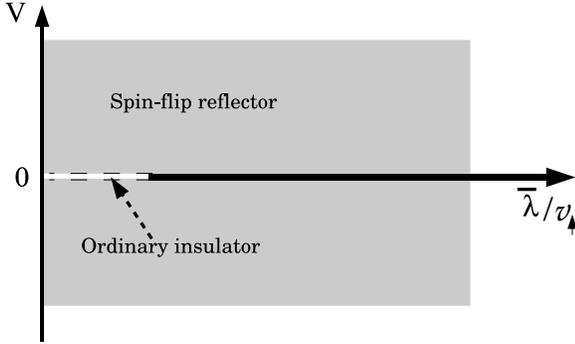


FIG. 2: Zero temperature phase diagram of a DW for $g_\perp > (g_\uparrow + g_\downarrow)/2$. Electron reflection is accompanied with spin reversal everywhere except at small $\bar{\lambda}$ and $V = 0$.

Considering stronger $\bar{\lambda}/v_\uparrow$, $g_\perp < (g_\uparrow + g_\downarrow)/2$ and $V = 0$ the zero temperature fixed point has $t = t' = r' = 0$ and $r_\uparrow = r_\downarrow = 1$. Near this fixed point, equation (16) yields:

$$\text{Re}[r'_\uparrow] \propto e^{(g_\uparrow + g_\downarrow)\xi}, \quad \text{Im}[r'_\uparrow] \propto e^{(g_\uparrow + g_\downarrow - 2g_\perp)\xi}. \quad (18)$$

The potential V , responsible for $\text{Re}[r'_\uparrow]$, is, therefore, irrelevant. This fixed point is stable for interactions obeying $g_\perp < (g_\uparrow + g_\downarrow)/2$ and unstable otherwise, thereby taking the system to the spin-flip reflector fixed point. For general finite V , the interactions determine whether the fixed point has $|r| = 1$, for $g_\perp < (g_\uparrow + g_\downarrow)/2$, or $|r'_\uparrow| = \sqrt{v_\uparrow/v_\downarrow}$ when $g_\perp > (g_\uparrow + g_\downarrow)/2$.

We can provide a physical explanation for the behavior of the DW shown in Figure 2 for small $\bar{\lambda}$. If the DW's transverse field ($\bar{\lambda}$) is zero, the renormalized reflection amplitudes r_\uparrow and r_\downarrow tend to 1 and -1, respectively. There is no reflection with spin flip as there is no spin-flip scattering term. Now consider an incident electron from the left with spin up. What is the probability amplitude of a spin-flip reflection for small $\bar{\lambda}$? There are two processes

to first order both in the interactions g and r'_\uparrow : (i) the electron is Bragg reflected back, in the \downarrow -spin channel, by the Friedel oscillations of spin-down electrons with a probability amplitude $g_\perp r'_\uparrow$; (ii) the electron is first reflected by the DW with amplitude r_\uparrow , then it is spin-flip back-scattered by the \uparrow -spin Friedel oscillation with an amplitude $g_\perp r'^*_{\uparrow}$, and finally reflected by the DW with amplitude r_\downarrow . Since $r_\uparrow r_\downarrow = -1$ and r'_σ is pure imaginary, process (ii) has exactly symmetrical amplitude to process (i). So, for $V = 0$, the two processes interfere destructively, resulting in the absence of spin-flip reflection. If $V \neq 0$, the amplitudes in processes (i) and (ii) acquire different phases and no longer cancel each other. In this case there is a reflection with spin reversal, with amplitude proportional to g_\perp , and if $g_\perp > \frac{1}{2}(g_\uparrow + g_\downarrow)$ the system will flow to the spin flip reflector phase. A strong $\bar{\lambda}$ leads to the fixed point with $r_\uparrow = r_\downarrow = 1$ and in this case there is no destructive interference.

B. Magnetic tip

The derivation of the RG equations for the magnetic tip problem follows exactly the same method as for the DW. In fact, the scaling equations for this case can simply be written down by inspection of the same Feynman diagrams:

$$\begin{aligned} \frac{dt_\sigma}{d\xi} &= 2g_\sigma |r_\sigma|^2 t_\sigma + 2g_{-\sigma} r_{-\sigma}^* r'_\sigma t'_\sigma \\ &+ 2g_\perp (r_{-\sigma}^* r'_\sigma t_\sigma + r_{-\sigma}^* r_\sigma t'_\sigma), \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{dt'_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 t'_\sigma + r_{\sigma}^* r'_\sigma t_\sigma) \\ &+ g_{-\sigma} (r_{-\sigma}^* r'_\sigma t_{-\sigma} + |r_{-\sigma}|^2 t'_\sigma) \\ &+ g_\perp (2r_{-\sigma}^* r'_\sigma t'_\sigma + r_{\sigma}^* r_\sigma t_{-\sigma} + r_{\sigma}^* r_{-\sigma} t_\sigma), \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{dr_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 r_\sigma + r_{\sigma}^* t_\sigma^2 - r_\sigma) \\ &+ g_{-\sigma} (r_{-\sigma}^* t'_\sigma t'_{-\sigma} + r_{-\sigma}^* r'_{-\sigma} r'_\sigma) \\ &+ 2g_\perp (r_{\sigma}^* r'_{-\sigma} r_\sigma + r_{\sigma}^* t_\sigma t'_{-\sigma}), \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{dr'_\sigma}{d\xi} &= g_\sigma (|r_\sigma|^2 r'_\sigma + r_{\sigma}^* t'_\sigma t_\sigma) \\ &+ g_{-\sigma} (|r_{-\sigma}|^2 r'_\sigma + r_{-\sigma}^* t_{-\sigma} t'_\sigma) \\ &+ g_\perp (r_{\sigma}^* r_{-\sigma} r_\sigma + r_{-\sigma}^* r_{\sigma}^2 + r_{\sigma}^* t_{-\sigma} t_\sigma \\ &+ r_{\sigma}^* t'_\sigma t'_{-\sigma} - r'_\sigma). \end{aligned} \quad (22)$$

The relations between the scattering amplitudes which follow from the Wronskian theorem are:

$$|r_\sigma|^2 + |t_\sigma|^2 + \frac{v_{-\sigma}}{v_\sigma} (|r'_\sigma|^2 + |t'_\sigma|^2) = 1, \quad (23)$$

$$r_\sigma^* r'_{-\sigma} + t_\sigma^* t'_{-\sigma} + \frac{v_{-\sigma}}{v_\sigma} (r_\sigma'^* r_{-\sigma} + t_\sigma'^* t_{-\sigma}) = 0, \quad (24)$$

$$r_\sigma^* t'_{-\sigma} + t_\sigma^* r'_{-\sigma} + \frac{v_{-\sigma}}{v_\sigma} (r_\sigma'^* t_{-\sigma} + t_\sigma'^* r_{-\sigma}) = 0, \quad (25)$$

$$\text{Re}[r_\sigma^* t_\sigma] + \frac{v_{-\sigma}}{v_\sigma} \text{Re}[r_\sigma'^* t_\sigma'] = 0, \quad (26)$$

$$v_\sigma t'_{-\sigma} = v_{-\sigma} t'_\sigma, \quad v_\sigma r'_{-\sigma} = v_{-\sigma} r'_\sigma. \quad (27)$$

The initial scattering amplitudes are the (non-interacting) solutions to (1):

$$t_\sigma = \frac{v_\sigma [v_{-\sigma} + i(V - \lambda')]}{[v_\sigma + i(V + \lambda')] \cdot [v_{-\sigma} + i(V - \lambda')] + \lambda^2} = r_\sigma + 1, \quad (28)$$

$$t'_\sigma = \frac{i\lambda v_\sigma}{[v_\sigma + i(V + \lambda')] \cdot [v_{-\sigma} + i(V - \lambda')] + \lambda^2} = r'_\sigma. \quad (29)$$

In this problem, the zero temperature fixed point is entirely determined by the interactions: if $g_\perp < (g_\uparrow + g_\downarrow)/2$ the system will flow to $r' = 0$, $|r| = 1$; if $g_\perp > (g_\uparrow + g_\downarrow)/2$ the system will flow to $r = 0$, $|r'_\uparrow| = \sqrt{v_\uparrow/v_\downarrow}$.

IV. DISCUSSION AND SUMMARY

Estimates of the g couplings have been made for the Hubbard model in a magnetic field¹⁶, where the inequality $g_{2\perp} > \frac{1}{2}(g_{2\uparrow} + g_{2\downarrow})$ was obtained in the strong coupling regime ($U > t$). Using the results for the renormalized Fermi velocities¹⁶ it is seen that a regime with $g_\perp > \frac{1}{2}(g_\uparrow + g_\downarrow)$ is possible, as well as the opposite inequality.

In the spin-flip reflector fixed point, the spinor wave function for an incident spin majority electron from the left has the asymptotic behavior:

$$\psi(z < 0) \simeq \begin{pmatrix} e^{ik_\uparrow z} \\ r'_\uparrow e^{-ik_\downarrow z} \end{pmatrix}, \quad (30)$$

for which the spatial distribution of the spin vector obeys

$$r'_\uparrow e^{-i(k_\uparrow + k_\downarrow)z} = e^{i\phi(z)} \tan \frac{\theta(z)}{2}, \quad (31)$$

where ϕ and θ are the spherical angles of the spin vector¹⁷. Since r'_\uparrow has modulus 1 and a finite phase,

then $\theta(z) = \pi/2$ implying that the conduction electron spin is normal to the wire and winds around it in "circularly polarized" fashion with azimuthal angle $\phi(z) = -(k_\uparrow + k_\downarrow)z$ for $z < 0$. For $z > 0$, $\phi(z)$ changes sign. Thus, the resulting spin density Friedel oscillations in the Luttinger liquid have such a structure and decay with distance as a power law.

Our RG method, being perturbative in the electron interactions, is valid when the latter are weak. But it allows for the scattering barrier to be treated exactly, whatever its strength, and hence predict the fixed point to which a particular barrier will flow once the interactions are turned on.

In summary, we considered the magneto-transport properties a magnetized one dimensional Luttinger liquid containing a localized disorder of the magnetization. We addressed two cases depending on the origin of this disorder: 1) a stray magnetic field of an STM tip disturbing the magnetization beneath it, or 2) a transverse domain wall (DW). Our theoretical analysis relies on a renormalization group treatment of the electron interactions that provides us with scaling equations for the transmission and reflection amplitudes as the bandwidth is progressively reduced. We find two $T = 0$ insulator fixed points: one in which the carriers are reflected in the same spin channel and another where the carrier's spin is reversed upon reflection leading to a finite spin current without a charge current. We also found the role of pure potential scattering in driving the DW to the spin-flip insulator phase when the transverse field of the DW is arbitrarily weak.

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