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Nonlinear Conductance of Nanowires – A Signature of Luttinger Liquid Effects?

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We analyze recent measurements of the room temperature current - voltage characteristics of gold nanowires, whose zero current conductance is quantized in units of $2e^2/h$. A faster than linear increase of current with voltage was observed at low voltages beginning from $V_c = 0.1$ V. We analyze the nonlinear behavior in terms of a dynamic Coulomb blockade of conducting modes and show that it may be explained as a Luttinger-liquid effect.

1. Introduction

Recently conductance quantization at room temperature was discovered [1] in metallic nanowires formed by breaking a contact between two metallic electrodes. Although the accuracy of the quantization in metallic nanowires as a rule is less then what can be achieved in ballistic nanocontacts formed in the two-dimensional (2D) electron gas of gated semiconductor heterostructures [2], the mere fact that plateaus appear at quantized values of conductance $G \simeq n(2e^2/h), (n = 1, 2, \ldots)$ shows that the electron transport through the nanowire is ballistic (the mean free path of charge carriers is larger than the length L and width W of the wire). In thin metallic nanowires the screening of ionic potentials by mobile charge carriers, which is very efficient in the bulk electrodes, could be considerably suppressed. Hence, the electrostatic potential felt by electrons moving along the wire becomes more smooth than is the case in bulk metals. In this picture the conductance quantization at room temperature can be understood within the framework of the "conductance is transmission" - picture — originally proposed by Rolf Landauer [3] — applied to a ballistic contact with almost perfect transmission. The approach of Landauer was subsequently developed by him and others into what we now know as the Landauer - Büttiker formalism, which is what is used below (see e.g. the review in Ref. [4]).

Suppression of screening in metallic nanowires implies in addition that the effects of electron-electron interactions could be important for the nature of electron transport through a nanocontact. It is well known that for 1D ballistic wires with a length that is effectively infinite, the interaction renormalizes the conductance quantum to a value which depends on the interaction strength (in the

Tomonaga-Luttinger liquid model the conductance per spin orientation is $K_\rho(e^2/h)$, where K_ρ is the charge correlation parameter of the Luttinger liquid [5, 6]). However, the conductance of a *finite* ballistic wire attached to leads with noninteracting electrons takes on integer values (in terms of the conductance quantum e^2/h) regardless of the interactions in the wire [7]. Therefore, the effects of electron-electron interactions are not revealed in the linear response regime for a finite ballistic (reflectionless) contact.

If the electron transport through a wire is not purely ballistic (i.e. there is a finite probability for electrons to be backscattered by impurity- or channel imperfections) the above picture is radically changed. In an effectively infinite 1D wire even arbitrarily small backscattering suffices to block the dc-current at vanishingly low voltages and temperatures [8, 9]. The effect is due to a strong enhancement of the bare backscattering amplitude by charge fluctuations at long distances. For a finite wire the renormalized barrier is also finite; the procedure of barrier renormalization should be stopped when a low-energy scale of order e^2/L is reached. Therefore, in a multichannel quantum wire the transmittivity of quantized modes will depend both on their bare transmission coefficients and on the strength of electron-electron interaction. The transmitting modes most sensitive to backscattering (the criteria will be given below) are converted to reflected modes by interaction effects. In other words the renormalized barrier is the actual barrier experienced by electrons propagating along the wire and this very barrier separates transmitting- from reflected quantized modes. The blockade imposed by the electron-electron interaction is gradually lifted by the bias voltage beginning from $V > V_c \simeq e/L$ resulting in a power-law $I - V$ characteristics.

Nonlinear $I - V$ characteristica for metallic nanowires in a regime where the linear conductance was quantized have recently been measured by Costa-Krämer *et al.* [10]. The following remarkable universal properties were observed:

(i) the characteristic crossover voltage $V_c \sim 0.1$ V to the nonlinear regime is sample-independent and anomalously small in comparison with the atomic energy scale ~ 1 eV set by the spacing of quantized mode energies in the contact [11]
(ii) the nonlinear character of the $I - V$ curves does not depend on the conductance in the zero voltage limit and is therefore independent of the number of conduction channels
(iii) the nonlinear contribution to the current is approximately proportional to the third power of the voltage

The purpose of this work is to show that all the above listed properties can be explained (at least qualitatively) if one associates the nonlinear current with the current produced by one (or possibly a few) — partly reflected — modes in a multichannel wire of interacting electrons. The current associated with this mode, which is small due to "Coulomb blockade" effects in the linear response regime, is strongly enhanced when the bias voltage is increased; eventually the voltage lifts the blockade and converts the reflected mode to an almost perfectly transmitting one. As we proceeed to elaborate this conjecture we recall first how the universal power-law dependence of current on voltage arises in the Luttinger liquid approach. To be concrete we consider a 1D model of interacting spinless electrons scattered by a local potential barrier.

2. Luttinger liquid model for nanowires – Weak interaction limit

For our purpose it is important to be able to describe a situation where the bare transmission probability can be close to — but not necessarily equal to — unity so that the conductance increases by essentially one quantum when the Luttinger liquid effects are suppressed (for example by a high bias voltage). This opening up of a new conduction channel is precisely what we propose may explain the nonlinearities in the measured $I - V$ curves. For arbitrary barriers the model can be treated analytically only for weakly interacting electrons when the effect of interaction can be reduced to a barrier renormalization produced by the formation of Friedel oscillations of the electron density near the barrier. The renormalized transmission coefficient is [12]

$$T^R(\varepsilon) = \frac{T_0 \left(\frac{\varepsilon - \varepsilon_F}{D_0}\right)^{2\gamma}}{R_0 + T_0 \left(\frac{\varepsilon - \varepsilon_F}{D_0}\right)^{2\gamma}}. \tag{1}$$

Here $T_0(R_0)$ is the bare transmission (reflection) coefficient ($T_0 + R_0 = 1$), ε_F is the Fermi energy, D_0 is an ultraviolet cutoff (see below), $\gamma \equiv \gamma_f - \gamma_b = [V_{ee}(0) - V_{ee}(2k_F)]/2\pi\hbar v_F$ is a dimensionless parameter which characterizes the strength of the electron-electron interaction, and $V_{ee}(q)$ is the Fourier transform of the interaction potential. Strictly speaking the parameter γ should be small for this approach to be valid ($\gamma \simeq e^2/2\pi\hbar v_F \ll 1$), but it is believed that Eq. (1) still holds qualitatively in the range $\gamma \sim 1$. In what follows we will regard γ as an input parameter.

The renormalized transmission coefficient (1) leads to an expression for the current when put into Landauer's formula. At zero temperature one finds

$$I(V) = \frac{e}{h} \int_{\varepsilon_F}^{\varepsilon_F + eV} d\varepsilon \, T^R(\varepsilon - \varepsilon_F) \equiv \frac{e}{h} F_\gamma \left(\frac{eV}{D_0} \left[\frac{T_o}{R_0} \right]^{\frac{1}{2\gamma}} \right). \tag{2}$$

We assume that the bare transmission amplitude, T_0, is energy independent near the Fermi level, so that the entire energy dependence of $T^R(\varepsilon)$ stems from the universal effects of barrier renormalization. The function $F_\gamma(x)$ has to be obtained numerically except in special cases (see below).

For a small renormalized transmission probability ($T^R \ll 1$) one can approximate Eq. (1) as

$$T^R(\varepsilon) \simeq \frac{T_0}{R_0} \left(\frac{\varepsilon - \varepsilon_F}{D_0} \right)^{2\gamma}. \tag{3}$$

The physical meaning of the high-energy cutoff D_0 in Eqs. (1) and (3) is the maximum energy transfered in a scattering event. For a strictly 1D wire with a local electron-electron interaction (Luttinger liquid) a natural cutoff is the bandwidth ε_F. If the interaction potential is characterized by a finite length scale l_s (a screening length) one has instead $D \sim \hbar v_F / l_s$. In a thin ($l_s \gtrsim W$) nanowire screening effects could be suppressed. In this case the Coulomb potential averaged over quantized transverse modes takes the form $V_c(x) \simeq e^2/\sqrt{x^2 + W^2}$ and the relevant high-energy cutoff is reduced to $D_0 \sim \hbar v_F / W$. The physics at scales $x \ll W$ can not be described by a long wavelength approximation and the Luttinger liquid-like approach is inappropriate.

The reasoning outlined above leads to an expression for the nonlinear tunneling current which takes the following form ($eV \ll D_0$)

$$I_{NL} \simeq \frac{ev_F}{W} \frac{T_0}{R_0} \left(\frac{eV}{D_0} \right)^{2\gamma + 1}. \tag{4}$$

We will model a nanocontact formed in conductance quantization experiments as an atomic-size constriction (characterized by a set of transmission coefficients for quantized modes) attached to a long ($L \gg W$) nanowire of width W (see Fig. 1). In an adiabatic model of a nanoconstriction [13] it is reasonable to assume that T_0 is *exponentially* close to zero or unity for all channels but one (or possibly a few). It should be stressed that in the range $\gamma \sim 1$ Eq. (4) holds even for T_0 close to unity due to a strong renormalization of the transmission amplitude. The crossover value for the ratio

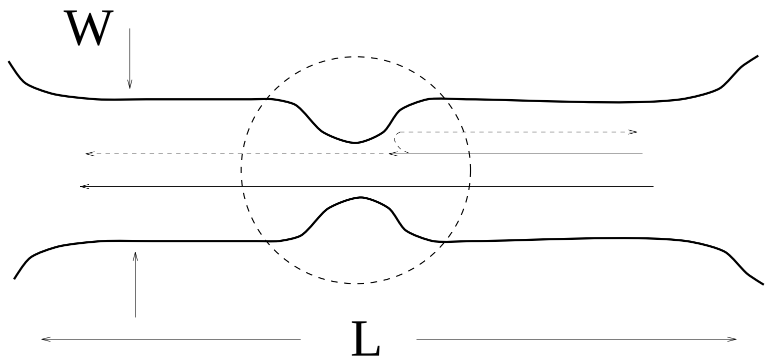

Fig. 1. Model of a nanowire contact. An atomic size constriction (within dashed circle) characterized by a set of transmission coefficients for quantized modes is attached to a long nanowire of of length L and width W. Most modes are either fully transmitted (lower arrow) or completely reflected (not shown) by the constriction. One, or possibly a few modes are partially reflected (upper arrow).

of unrenormalized coefficients T_0/R_0 that separates "under-the-barrier" from "over-the-barrier" transport is $(T_0/R_0)_c \simeq (L/W)^{2\gamma}$ which is much bigger than unity if $\gamma \sim 1$. If T_0 is close to one we should use the exact expression (1) rather than Eq. (3) when describing high voltage behavior.

Since the nonlinear contribution to the currents measured in Ref. [10] was fitted to a power law, V^q, with an exponent q statistically peaked at $q \simeq 3$, the specific value of interaction strength $\gamma = 1$ appears to be of most interest to us. In this special case the nonlinear conductance calculated from Eqs. (1) and (2) takes a simple analytic form

$$G(V) \equiv \frac{I(V)}{V} = \frac{e^2}{h}\left[1 - \frac{\arctan(x)}{x}\right], \; x = \frac{eV}{D_0}\sqrt{\frac{T_0}{R_0}} \tag{5}$$

Therefore, if the "Coulomb blockade" of the quenched tunneling mode is lifted, the nonlinear voltage dependence of the current goes over to the linear dependence expected for a perfectly transmitting mode. As for the modes which are propagating, "over-the-barrier" reflection only results in a small additive renormalization of the linear current.

Is it reasonable to trust the power law $I - V$ result even in cases when the coupling strength is not small? We think so; actually this is a quite general result for Luttinger liquid-like theories (see e.g. the reviews [14]). The analogous expression to Eq. (4) can be derived in the tunnel Hamiltonian model for the whole range of interaction strengths. However, in this approach one has to assume a small unrenormalized transmission coefficient($T_0 \ll 1$) to get analytic results [6, 15]. For our purpose it was important to describe scattering of interacting electrons by a barrier of arbitrary height, therefore we have used the approach elaborated in Ref. [12].

For a finite wire of length L renormalization of the transmission amplitude should be stopped at energies $\delta\varepsilon \equiv |\varepsilon - \varepsilon_F| \leq \Delta = 2\pi\hbar v_F/L$. In the weak coupling limit the low-energy cutoff Δ corresponds to the minimum-energy charge excitations in a finite wire (longitudinal energy discretization). In the case of strong interaction the cutoff depends on the plasmon velocity s and is $\Delta(s/v_F)^2 \sim e^2/L$ (see Section 3). When describing the current in a finite wire Eqs. (4) and (5) are valid for voltages $V > V_c \simeq \Delta/e$. At lower voltages the transmission coefficient is energy independent $T^R(\delta\varepsilon \sim \Delta) = (T_0/R_0)(W/L)^{2\gamma}$. The current produced by the quenched mode at low voltages, $V < V_c$, is linear and small even if the bare transmission amplitude is close to one,

$$I = \frac{e^2}{h}\frac{T_0}{R_0}\left(\frac{W}{L}\right)^{2\gamma} V, \qquad V < V_c \,. \tag{6}$$

2.1. Comparison between theory and experiment

Employing the above simple formulae for an explanation of the nonlinear conductance of metallic nanowires we first note that the height of the measured conductance steps [10] deviate somewhat from multiples of e^2/h (i.e. from perfect quantization). By associating the difference δG with the small current carried by Coulomb blockaded mode at low voltages we conclude from Eq. (6) that

$$\delta G/(e^2/h) \sim (T_0/R_0)(W/L)^{2\gamma} \,. \tag{7}$$

As discussed above the suppression of the current in this quenched mode is lifted by a sufficiently large applied voltage. This gives rise to a nonlinear contribution to the $I-V$ curves when $V \sim V_c$. It follows that the nonlinearity is due to a suppression of Luttinger liquid-like effects at high voltages. Since the nonlinear contribution to the current arises from the mode most sensitive to backscattering it does not depend on the number of conduction channels (see below). The universal character of the nonlinear contribution is due to the "long distance" origin of barrier renormalization; the relevant energy scale is either given by the total length of the nanowire L (if $V < V_C$) or by the "field" length $l_E = \hbar v_F/eV \gg W$.

Simple numerical estimations show that the above formulae allow us to explain the $I-V$ characteristica measured in Ref. [10]. First of all the crossover voltage observed in the experiment ($V_c \simeq 0.1$ V) can be readily obtained from Eq. (5) for metallic nanowires a few nanometers long. Since the numerical coefficient in the theoretical estimation of V_c is uncertain it is difficult to be more precise in the quantitative predictions. However, it is worthwhile to stress that the voltage in question is determined by the total length of the nanowire and not by the length of its narrowest part which provides the conductance quantization. Therefore the crossover to the nonlinear regime in this picture does not depend on uncontrollable factors like precisely how the constriction is formed.

From the experiments of Ref. [10] we conclude that the exponent $(2\gamma + 1)$ in Eq. (4) should be approximately equal to 3, hence $\gamma \simeq 1$. For Coulomb interaction $V_{ee}(0) \gg V_{ee}(2k_F)$ and one can estimate $\gamma \simeq (e^2/\pi\hbar v_F)\ln(L/W)$ which for reasonable values of L/W and v_F typical for metals is in amazingly good agreement with the value extracted from experiment. This numerical coincidence could be a strong argument in favour of our ideas if both theoretical predictions and the chosen "experimental" value of exponent ($q = 3$) in power-law $I-V$ curves were *quantitatively* reliable. Unfortunately there is no physical reason to believe that Eq. (1) is quantitatively correct in the range $\gamma \sim 1$. As for experiment [10], the distribution of the exponent q for different measurements, although peaked at $q = 3$, spreads out from 2 to 4. So instead of playing with numbers it would be more reasonable to look for *qualitative* features which can be easily either confirmed or ruled out by experiments.

The qualitative predictions which emerge from our picture are straightforward to extract from the theory. First, the larger the fluctuations of the conductance around its quantized value the bigger the nonlinear current. One can find some experimental confirmation of this prediction by examining Fig. 11 of Ref. [10]. (The fluctuations of conductance are obviously increasing as the number of conduction channels [modes] increases. So one can expect that the absolute value of the nonlinear current will increase as well. Such a tendency can indeed be observed in Figs. 11(a-d) of the cited paper). Next, it is evident that the contribution of the tunneling mode to the current when the Coulomb blockade is lifted by high voltages could at most be comparable to that of a freely

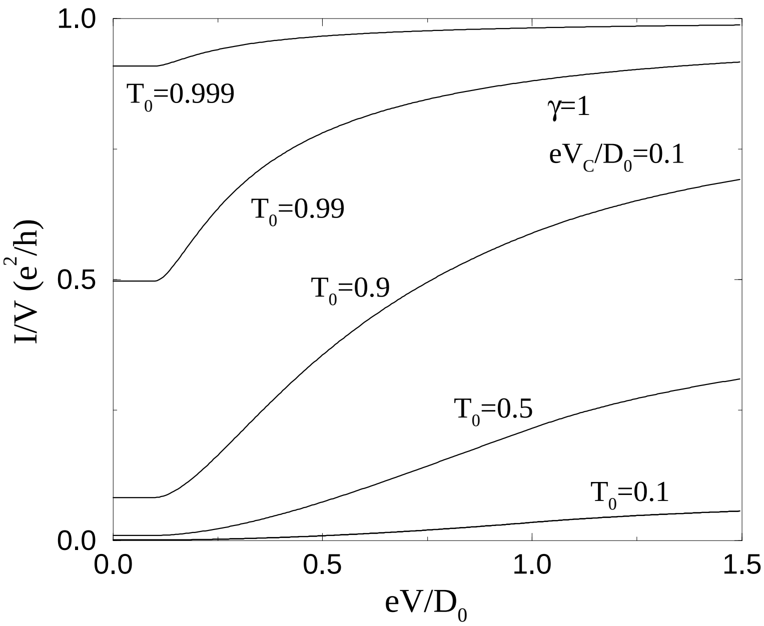

Fig. 2. Conductance $G(V) \equiv I/V$ calculated from the Tomonaga-Luttinger liquid model, Eqs. (5) and (6). It represents the contribution of a conductance channel in which electrons are partly reflected ($T_0 < 1$) from a potential barrier renormalized by electron-electron interactions, the strength of which is characterized by a parameter γ. The parameter $D_0 = \hbar v_F/W$ is a high energy cutoff associated with the time it takes for a charge excitation to spread across the width W of the wire, while $V_c \sim \hbar v_F/L$ is a low energy cutoff related to the finite wire length L.

propagating mode (for which $R_0 \ll T_0$). Therefore the *total* current of an n-channel constriction at saturation voltage (which is determined by a short distance length scale) should approach the *linear* current of an $(n+1)$ channel nanocontact.

Plots of the nonlinear conductance $G(V)$ calculated from Eq. (5) are shown in Fig. 2. Even for a very nearly ballistic mode with $T_0 = 0.9$ one can see that there is a significant (although not complete) restoration of transmittivity for voltages in a range relevant for the experiment. However for the quenched modes with small backscattering complete restoration is achieved at high voltages $V \sim V_s \simeq \sqrt{T_0/R_0}D_0 \gg D_0$, where the Luttinger liquid approach does not hold. In the theoretically reliable region of voltages $eV < D_0$ the current carried by each individual (quenched) mode is a few times smaller than for a perfectly transmitting channel. In the experiments [10], on the other hand, the conductance is increased by approximately one quantum unit already in the low voltage region. In our picture the observed conductance enhancement could be explained if more than one suppressed mode contribute to the nonlinear current.

2.2. Spin degrees of freedom and multichannel effects

Although the above formulae pertain to the case of spinless fermions, the inclusion of spin does not change the situation radically. As was shown in Ref. [12], adding the spin degree of freedom only results in an obvious additional factor of 2 in the conductance for the case of a smooth interaction potential, $V_{ee}(2k_F) \ll V_{ee}(0)$. In a general case the transmission probability in Eq. (1) should be replaced by [12]

$$T_0 \left(\frac{\varepsilon}{D_0} \right)^{2\gamma} \longrightarrow T_0 \left(1 + 2\gamma_b \ln \frac{D_0}{|\varepsilon - \varepsilon_F|} \right)^{3/2} \left| \frac{\varepsilon - \varepsilon_F}{D_0} \right|^{2\gamma_f - \gamma_b} \tag{8}$$

It is evident that the additional logarithmic energy dependence of the transmission probability can not change our estimations in an essential way.

A less evident point in connection with the above purely 1D considerations is how they are affected by multichannel effects. In reality a metallic nanowire may only be able to support a few conductance channels at its most narrow part where the width is of the order of a few Fermi wavelengths. The wire gets much broader, however, away from this region and many conduction channels are supported. To discuss this complication we will first argue that even though the potential barriers that define the wire are due to atoms the adiabatic approximation applies.

In principle a rearrangement of the atomic structure as the the wire is extended could lead to a step-like behavior of the conductance, but there is no reason to expect such conductance steps to be quantized in units of e^2/h. Experiments, on the other hand, have definitely demonstrated such a conductance quantization (statistically) of metallic nanowires (see e.g. the review [16]). Moreover, the conductance staircase measured by the STM technique at room temperature [10] looks very similar to the one found in 2D electron gas systems. In particular, the accuracy of the quantization is quite high (within 10% for the first step) and all plateaus were observed (up to $n = 6-8$) with no indication of suppressed conductance steps at $n = 2, 4, 7$ as predicted for a cylindrically symmetric ballistic microcontact. This means that for the selected measurements, and when the conductance is quantized, the motion of electrons through the constriction can be considered adiabatic; so a simple “2D”-model (with no degenerate transverse modes) of an adiabatic contact [13] may be quite adequate for our purposes.

In the adiabatic approach to electron transport through a quantum contact the transverse (fast) and longitudinal (slow) motion of electrons through the contact are decoupled when the Schrödinger equation is solved. The kinetic energy due to the transverse motion then appears as a potential barrier for the longitudinal motion which adds to the smooth potential from the constriction itself. The smooth effective potential is different for different modes and has a maximum where the contact diameter has its minimum. Since the energy level spacing imposed by transverse momentum (mode) quantization is of the order of the Fermi energy, one can use the following simple classification of modes: those with energies above the barrier are transmitting modes, those below the barrier reflected modes. As we have seen already this classification should be changed for interacting electrons, where the renormalized potential rather than the bare potential is relevant. One can infer from Eq. (1) that all modes for which $T_0/R_0 \ll (L/W)^{2\gamma} \gg 1$ are quenched by the strong enhancement of backscattering. It is known that for smooth potentials over-the-barrier reflection decreases exponentially with increasing energy of the incident particle. Hence it is quite natural to assume that only the last mode (the one with energy closest to the potential barrier maximum) will change its character by renormalization effects and be converted from a transmitting to a reflected mode. All other modes keep their unrenormalized character; they are either transmitting modes (over-the-barrier) or totally reflected modes (under-the-barrier).

Let us now ask the question how a transmitting mode with $T_0 = 1$ is affected by inter-mode (inter-channel) *forward* scattering in a multichannel wire. The answer is that in an effectively infinite multichannel Luttinger-liquid wire the Coulomb interaction smears out the quantized conductance plateaus so that in the strong interaction limit the conductance is described by almost smooth function of the wire width (or of the Fermi energy) [18]. This effect is entirely due to the properties of plasmon excitations at long distances. For a wire of *finite* length connected to large reservoirs the dc current will be determined by the properties of plasmons in the reservoirs [18, 7]. So in experiments, where the reservoirs are massive metals, the conductance of a perfect multichannel wire of interacting electrons will be quantized exactly as if the electrons in the wire were noninteracting.

Now we turn to another question: Could the Coulomb-blocked mode, which is reflected by the renormalized barrier, affect the propagation of the transmitting modes? In the case of weak interaction the answer is that it definitely cannot. First of all the presumed smooth bare potential can not induce intermode scattering. Therefore each mode with $R_0^j \neq 0$ (j is the mode index) sets up its own Friedel oscillations when reflected by the potential. The corresponding induced barriers are characterized by different oscillation periods $\lambda(\varepsilon = \varepsilon_F^j)$, where ε_F^j is the "Fermi energy" in the j:th channel. (The channel dependence comes from the fact that ε_F^j is measured from the quantized transverse energy corresponding to the j:th mode; it is the bandwidth of the j:th mode). Interaction enhanced backscattering arises only for electrons in the same mode or subband [since the corrections to bare transmission amplitude is proportional to $R_0^{(j)}$]. Particles with energies in the vicinity of ε_F^j are phase locked to the Friedel oscillations and thus strongly backscattered. The influence of this effect on the transmittivity of quantized modes have already been studied above. Perfectly transmitting modes ($R_0^{(j)} = 0$) can be scattered by the potential due to Friedel oscillations of the charge density in the quenched channel. However, this scattering could lead at most to nonsingular (length independent) contributions. In the lowest order of perturbation theory the corresponding induced backscattering amplitude $|r^{(j)}| = \pi\gamma|r_0|$ (r_0 is the bare backscattering amplitude of quenched mode) is small if $|r_0| \ll 1$.

To put it differently, transmitting modes will propagate freely even in an interacting electron system irrespective of possibly strong backscattering in the other modes. Is it possible to extend this claim to the strongly interacting case? To clarify this point we switch now to the case of strong coupling, where the picture of Luttinger liquid transport through a constriction will look slightly different.

3. The strong coupling regime

In the strong coupling regime the suppression of tunneling for repulsively interacting electrons can be easily understood in terms of a Wigner crystal (or charge density wave) pinned by impurities [9]. The quantum depinning of a Wigner crystal is accompanied by propagation of plasmon excitations along the wire. The gapless sound-like spectrum of 1D plasmons results in a logarithmically divergent contribution of plasmons to the tunnel action. The infrared cutoff (at small momenta) appears at scales determined by temperature or voltage. However, in the case of a *finite* wire (either isolated or connected to 3D leads) one gets the additional natural energy scale $\Delta_s = \hbar s/L$, where s is the plasmon velocity. For an isolated wire (or a ring, see e.g. [17]) this scale is provided by the discrete spectrum of energy levels; for a wire connected to 3D leads a cutoff appears due to the fact that the excitation spectrum of 3D plasmons has a gap and does not give rise to infrared divergencies.

As in the case of weak coupling we will associate the nonlinear contribution to the current with the interaction-quenched mode. If the interaction is not weak one can treat the Luttinger liquid transport through a barrier analytically only in the limit of a strong barrier. So our consideration could be applied strictly speaking to the tunneling regime of conductivity. We start with a purely 1D picture (a single mode channel). Now the current at temperatures effectively equal to zero (for metals this temperature interval includes room temperature) is given by

$$I(V) = \frac{e}{h}\int_0^{eV} d\varepsilon D(\varepsilon), \tag{9}$$

where

$$D(\varepsilon) = \left(\frac{\varepsilon}{V_p}\right)^{\frac{2}{\alpha}}, \quad \alpha = \frac{v_F}{s} \ll 1 \,. \tag{10}$$

is the tunneling probability associated with the excitation of long wavelength plasmons [9] (notice that in this reference the exponent in the formula for the tunneling probability contains an incorrect numerical factor). In Eq. (10) V_p is the pinning potential which for strong repulsion ($\alpha \ll 1$) can be put equal to the height of the potential barrier [9].

At high voltages $eV > \Delta_s/\alpha$ (however still much lower than the height of the barrier) the current is nonlinear

$$I_{NL}(V) \simeq \frac{e}{h}\alpha V_p \left(\frac{eV}{V_p}\right)^{\frac{2}{\alpha}} \,. \tag{11}$$

Notice that we omitted terms of order unity in the exponents of Eqs. (10) and (11) since the accuracy of calculations in the strong coupling limit ($\alpha \ll 1$) is not so good as to make it meaningful to keep them.

At small voltages the tunnel action should be cut off at the energy scale Δ_s and the tunnel current becomes linear

$$I = \frac{e^2}{h}\left(\frac{\Delta_s}{V_p}\right)^{\frac{2}{\alpha}} V, \;\; eV \ll \Delta_s/\alpha \,. \tag{12}$$

To get the kind of nonlinear $I - V$ characteristics measured in Ref. [10] one should put $\alpha \simeq 2/3$ in Eq. (12). Again, this value of the correlation parameter is not sufficiently small to justify a quantitative comparison between theory and experiment.

An equation for the nonlinear current [analogous to Eq. (11)] valid in the required range of interaction strength can be derived in the tunnel Hamiltonian approach [15]. The corresponding formula for a *finite* wire connected to 1D reservoirs of *noninteracting* electrons (for ballistic transport an analogous model was considered in Ref. [7]) takes the form

$$\begin{aligned} I(V) &= \frac{4e}{\hbar}\frac{|t|^2}{\Delta_s}\int_0^\infty dx \cos\left\{\frac{eV}{\Delta_s}x - \frac{2}{\alpha}\arctan\left(\frac{\Lambda}{\Delta_s}x\right) + \left(\alpha^{-1}-1\right)\left[\pi + 2si(x)\right]\right\} \\ &\times \left[1 + \left(\frac{\Lambda}{\Delta_s}x\right)^2\right]^{-\frac{1}{\alpha}} \exp\left\{2\left(\alpha^{-1}-1\right)\left[C + \ln(x) - ci(x)\right]\right\}, \end{aligned} \tag{13}$$

where t is the tunneling matrix element which is assumed to be small, $C \simeq 0.577\ldots$ is the Euler constant, Λ is the ultraviolet cutoff ($\Lambda \gg \Delta_s,\ \Lambda \gg eV$), and the integral sine (cosine) function is defined as follows

$$\begin{pmatrix} si \\ ci \end{pmatrix}(x) = -\int_x^\infty dt \frac{\begin{pmatrix} \sin \\ \cos \end{pmatrix}(t)}{t} \tag{14}$$

For $\Delta_s \to 0$ (infinite wire) Eq. (13) yields the well-known result for Luttinger liquid transport of a power-law $I - V$ dependence [6, 15]

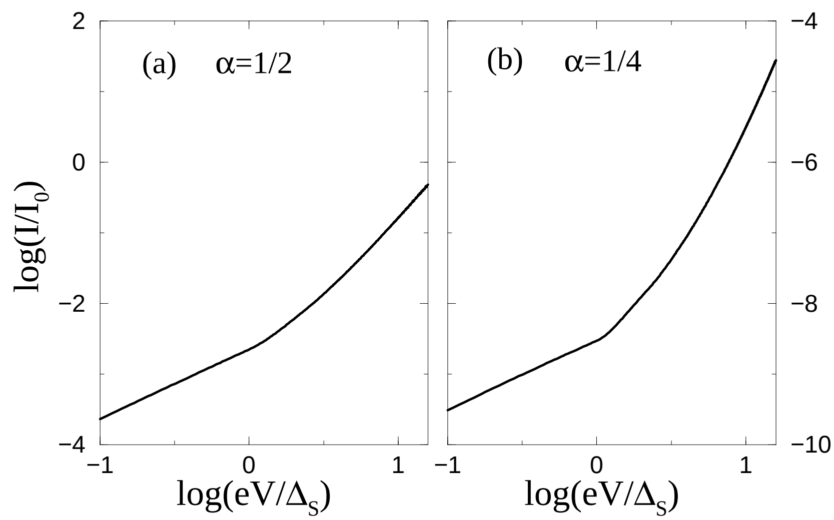

Fig. 3. Log-log plots of the nonlinear tunneling current (Eq. 13) vs. gate voltage for a finite quantum wire connected to reservoirs of non-interacting electrons. Here $I_0 = (e/h)|t|^2/\Lambda$, where t is the tunneling matrix element, $\Lambda \sim \hbar v_F/W$ is a high-energy cut-off related to the wire width W; $\Delta_s = \hbar s/L$ (s is the plasmon velocity) is an energy scale associated with the discrete energy spectrum and hence the finite wire length L. In the calculation $\Lambda/\Delta_s = 50$. For small voltages, $V < \Delta_s/e$, the current increases linearly with voltage. For higher voltages the current depends on the interaction parameter α and the slope in the figure approaches the infinite-wire length value $2/\alpha - 1$ of Eq. (15).

$$I_{NL} \simeq \frac{e}{h}\frac{|t|^2}{\Lambda}\left(\frac{eV}{\Lambda}\right)^{2\alpha^{-1}-1} . \tag{15}$$

For a finite wire the integral in Eq. (13) has to be evaluated numerically. The corresponding $I-V$ curves are plotted in Fig. 3. From this figure one can see that at low voltages the current grows *linearly with voltage* and is small. Beginning from $V \sim V_{c1} \simeq \Delta_s/e$ the tunnel current increases rapidly, eventually reaching the asymptotic value Eq. (15) at $V > V_{c2} \simeq \Delta_s/e\alpha$.

As we have noticed already the above formulae describe the tunnel current and strictly speaking they can not be used for explanaining the nonlinear $I-V$ behavior measured in the experiment [10] (where the observed increase of conductance was of the order of the conductance quantum). However, in order to analyze the phenomenon in terms relevant for the strong coupling regime one may put $|t| \sim \Lambda \sim \hbar v_F/W$. For a comparison with experiment it is reasonable to use the expression [analogous to Eq. (15)], which describes the nonlinear current of spin-1/2 electrons. The corresponding formula can be readily obtained from Eq. (15) by the substitution $2\alpha^{-1} \to \alpha_\rho^{-1}+\alpha_\sigma^{-1}$, where α_ρ (α_σ) is the Luttinger liquid correlation parameter for the charge (spin) sector). For an $SU(2)$ invariant spin interaction (relevant here) $\alpha_\sigma = 1$. The correlation parameter in the charge sector can be expressed in terms of the Fourier transform of the forward scattering interaction potential (see e.g. Ref. [18])

$$\alpha_\rho^{-2} = 1 + \frac{2V_{ee}(0)}{\pi\hbar v_F} . \tag{16}$$

For a cubic $I-V$ behavior $\alpha_\rho \simeq 1/3$ and the corresponding value of the dimensionless interaction strength is $\gamma \simeq 2$, which is larger by a factor 2 than the value extracted in the weak coupling limit. All numerical estimations performed for the weak coupling limit can be equally well reproduced using formulas corresponding to the strong coupling regime (obviously with the same uncertainties in numerical coefficients of order one). As a result one has to admit the evident failure to *quantitatively* describe nonlinear transport in metallic nanowires using naively a Luttinger liquid model. However it is suggestive that both limits yield reasonable values for the interaction parameters (of order one, which is typical for metals).

The advantage of looking at our problem in the strong coupling limit is that it allows us to consider the multichannel case analytically. At first we assume that the current through the nanoconstriction is due to tunneling. Then the problem is reduced to describing the propagation of charged excitations (plasmons) in a multichannel wire. For this case (here we again for simplicity will consider spinless fermions) it is known [18] that in the limit of a large number of channels $N_\perp \gg 1$ the plasmon velocity s is given by

$$s = \sqrt{\sum_{j=1}^{N_\perp} n_j \frac{e^2}{m}} \ , \tag{17}$$

where n_j is the electron density in the j-th channel. Hence it is determined by the *total* density of electrons in a multichannel wire. It is this very density (which approaches the bulk density of electrons in the vicinity of the electrodes) and not the smaller density of electrons in the few quantized modes at the center of the contact which will determine the actual value of the correlation parameter α in Eq. (11). Therefore, the exponent in the power-law type $I-V$-curves should be independent of the number of conductions channels as was claimed in Ref. [10].

A situation where the number of channels varies along the wire can be modelled by assuming the plasmon velocity to be a function of position $s \to s(x) = s_0 f(x)$. Straightforward calculations analogous to those performed in Ref. [9] (see also [17]) yield

$$D(\varepsilon) = \exp\left(-\frac{2}{\alpha_0}\int_\varepsilon^{V_p} dx \frac{f(x)}{x}\right) \ . \tag{18}$$

According to Eqs. (9) and (18) the nonlinear conductance of a general multichannel wire will depend on the actual shape of the wire. However, if the function $s(x)$ is smooth, the tunneling probability still has a local energy dependence,

$$D(\varepsilon) = \left(\frac{\varepsilon}{V_p}\right)^{\frac{2}{\alpha(\varepsilon)}} \ , \quad \alpha(\varepsilon) = v_F/s(\varepsilon) \ . \tag{19}$$

Therefore to get in this approach a sample-independent exponent describing the nonlinear conductance observed in Ref. [10] one has to assume that the nanowire has an almost uniform shape far from the constriction (this is exactly the model for a nanocontact used here, see Fig. 1).

The last point we would like to discuss is how the mode quenched by interaction effects influences the transmitting modes. In the strong coupling regime the assumption of perfect transmission means that the corresponding Wigner crystals can slide freely (without distortions) along the channels. If the interaction potential is smooth and one can neglect effects of backscattering [which are proportional to the exponentially small interaction constant $V(2k_F)$] the degrees of freedom associated with perfectly transmitting modes decouple from the rest. Their dynamics is linear and can be treated exactly [7] for an adiabatic constriction. One may conclude that as long as the backward scattering part of the interaction potential is irrelevant, perfect conductance quantization can not be spoilt by quenching of reflected modes.

4. Conclusions

Summing up all facts in favour of the above picture, we conclude that the Luttinger liquid-like effects could be a reasonable explanation of the nonlinear conductance measured in metallic nanowires. If so the experiment by Costa-Krämer *et al.* [10] can be regarded as one of only a few highly important experiments (see also [19, 20]) where the fundamental properties (Kane-Fisher effect) of Luttinger liquid-like states of strongly correlated electrons were observed.

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